INJECTION OF A PLASMA CLOUD IN A TRANSVERSE MAGNETIC FIELD

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16. Abstract

A qualitative description is given of the evolution of a dense plasma cloud, which is injected into the magnetosphere perpendicular to the magnetic field force lines. Characteristic times are evaluated for the free expansion and diffusion of the magnetic field into the cloud. Hydromagnetic instability questions are considered for the cloud surface. This instability is related to the surface curvature and the passage of the cloud around the magnetosphere plasma at the initial stage of magnetic field diffusion into the cloud. The effects of pressure anisotropy, which is caused by the passage of the cloud along the magnetic field force lines, are studied on the drift, and current instabilities are studied in the case of subsequent spreading of the cloud along the magnetic field. New instability modes are found, which significantly affect the pressure anisotropy in an inhomogeneous plasma, or in the presence of a flux of particles in the plasma.

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ANNOTATION

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1. Qualitative Discussion of the Cloud Dynamics

Investigation of the behavior of plasma clouds (both charged and quasi-neutral) in a transverse magnetic field is of current interest in connection with the use of artificial perturbation for studying cosmic plasma [1 - 8]. /3*

This article considers the injection of a quasi-neutral plasma cloud in a transverse magnetic field. Here, the following cloud parameters are used (CGS system): overall cloud velocity along the injection axis $x, \sim v^{\circ} \sim 10^{7}$, dispersion angle $\theta^{\circ} \sim 15^{\circ}$ (see Figure 1), dispersion velocity along the z-axis $v_{\circ} = v_{z} \sim v^{\circ} \sin\theta \sim v^{\circ}\theta \sim 4 \cdot 10^{6}$, the initial dispersion velocity along the y-axis, particle temperature $T_e \sim T_i \sim T \sim 10\,ev \sim 10^{5}$, speed of sound $v_s = \sqrt{T_e/M_i} \sim 3 \cdot 10^{6} \frac{1}{\sqrt{x}}$, $x = M_i/M$, M is the mass of a hydrogen atom, $v_{T_i} \sim v_s \sim 3 \cdot 10^{6}/\sqrt{x}$, $v_{T_e} \sim 10^{8}$. The injected cloud volume is $V_0 \sim 1\,л = 10^{3}$, and the energy in this volume is $\mathcal{E}_0/x \sim 1$ joule = 10^7. The number of particles in the cloud is

$$N = N_i = N_e = \mathcal{E}_0 / M_i \frac{v^{\circ 2}}{2} = \frac{\mathcal{E}_0}{x} \frac{2}{M v^{\circ 2}} \sim 10^{17}$$

The initial cloud density is

$$n_0 = n_{0i} = n_{0e} = \frac{N}{V_0} \approx 10^{14}$$

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*Numbers in the margin indicate pagination in the original foreign text.

The ratio of the gas-kinetic pressure of the plasma to the magnetic field pressure is /4

$$\beta = \frac{8\pi \sum_i nT}{B^2} \sim 4\, v_s^2 / v_A^2 \,, \tag{1.1}$$

where $v_A = B/\sqrt{4\pi \rho_m}$ is the Alfven velocity ($\rho_m = NM$). At the initial moment of time, $\beta = \beta_0 \sim 5 \cdot 10^5$. The injection time $t_u \sim 10$ μsec = 10^{-5}. The magnetic field strength is $B \sim 0.3$. The ratio of the dynamic pressure of the cloud, which moves with a velocity v_j along the j-axis, to the magnetic pressure is:

$$\beta_j = \frac{4\pi \rho_m v_j^2}{B^2} = \frac{v_j^2}{v_A^2} \tag{1.2}$$

a) Free expansion stage of the cloud

The magnetic field does not affect the expansion of the cloud as long as $\beta > 1$. As a result, the final cloud dimensions at the stage of free expansion ($\beta \gtrsim 1$) are large compared to the initial dimensions, so that the initial cloud dimensions can be neglected. Thus, the cloud density decreases with time according to the equation:

$$n = \frac{N}{V} \sim \frac{N}{v_s v_0^2 t^3} \,, \tag{1.3}$$

since the cloud expansion velocity is of the order of v_0 along the y and z-axes, and of the order of v_s along the x-axis (in a coordinate system at rest with the center of inertia of the cloud). Since $v_0 > v_s$, cloud expansion is stopped first by the action of the magnetic field with respect to 0 and x. This occurs at $t \sim t_1$, when $\beta \sim 1$. Thus, according to Equation (1.1):

$$n_1 = \frac{B^2}{16\pi T} \sim 10^8 \tag{1.4}$$

Substituting Equation (1.4) in (1.3) yields: /5

$$t_1 \sim \left(\frac{N}{n_1 v_s v_0^2}\right)^{1/3} \sim 3 \cdot 10^{-4} \varkappa^{\frac{1}{6}} \tag{1.5}$$

Here, the cloud dimensions are of the same order in all directions:

$$L_1 \sim v_s t_1 \sim 10^3 \varkappa^{-\frac{1}{3}}. \tag{1.6}$$

We note that this discussion [see Equation (1.3)] is correct for these parameters only if:

$$t_1 > t_0 \, v^0 / v_s \, . \tag{1.7}$$

when it is possible to neglect the dimensions which the cloud has at the moment the injection ends, compared with its dimensions at time t.

b) Effect of the magnetic field on the cloud motion at the stage when the magnetic field is diffusing into the cloud

Further cloud motion perpendicular to the magnetic force lines is determined by the magnetic field (for $\beta \lesssim 1$). Here, the cloud shape can be quite different, depending on the parameter $\varkappa$.

1. Motion of a light plasma cloud ($\varkappa \sim 1$).

Expansion of the cloud ceases along x and y almost at the same time (for t $\simeq$ t), as indicated by Equations (1.1) and (1.2) for $\varkappa \sim 1$, when the velocity $v_y \cdot v_0 \sim v_s$, that is, when $\beta_y \sim \beta$. The inertia of the cloud will cause it to continue to expand at time ($\sim t_1$) perpendicular to the magnetic field (and simultaneously along the field), so that the parameter β becomes less than unity, and the magnetic field starts to collapse the cloud perpendicular to the force lines with a velocity $\sim v_A$. The cloud density increases faster from this two-dimensional compression than it decreased, because of the spreading of the plasma along the field, so that the density soon (in a time $\sim t_1$) again reaches a value $n \geq n_1$, when $\beta \geq 1$, and so forth. In this manner, the cloud will oscillate with cylindrical symmetry relative to the z-axis, with a continually decreasing radius. Here, the cloud density will remain almost constant ($n \sim n_1$). The average /6 cloud radius and length will vary with time as follows. Since the magnetic field does not affect the cloud expansion along the force lines,

$$L_z \sim L_1 + v_0\tau, \tag{1.8}$$

where t is time measured from the end of the free expansion of the cloud. Since the cloud volume deforms as $V \sim y^2 z$ in two-dimensional compression, and remains almost constant (N = const, $n \sim n_1$),

$$L_y \sim L_x \sim \frac{L_1^{3/2}}{\sqrt{L_1 + v_0\tau}}, \quad v_y \sim v_x \sim \frac{v_0 L_1^{3/2}}{2(L_1 + v_0\tau)^{3/2}} \tag{1.9}$$

Diffusion of the magnetic field into the cloud occurs simultaneously with the collapse of the cloud. The diffusion is related to the dissipation of the currents induced in the cloud, which compensate for the external magnetic field in the cloud volume. The cloud compression process perpendicular to the field ends after a time $\tau \sim \tau_d$, during which the magnetic field completely diffuses into the cloud. At this moment, the transverse cloud dimensions will be:

$$L_{y_2} \sim L_{x_2} \sim \frac{L_1^{3/2}}{\sqrt{L_1 + v_0\tau_d}} \tag{1.10}$$

from Equation (1.3). On the other hand, it is known that in time τ_d, the magnetic field will diffuse into a plasma perpendicular to the magnetic force lines to a depth /7

$$L_\perp \sim \sqrt{D_\perp \tau_d}, \tag{1.11}$$

where $D_\perp = \frac{c^2}{4\pi\sigma_\perp} = \frac{\nu_{ef} c^2}{\omega_{pe}^2}$ is the diffusion coefficient (magnetic viscosity) of the field, σ_1 is the effective collision frequency of the cloud electrons, and ω_{pe} is the Langmuir frequency of the electrons. In the case of the collision-controlled dissipation, ν_{ef} is determined by Coulomb collision of the electrons with the electrons and ions of the cloud, and also by the neutrals in the ionosphere. (For the parameters used here, the neutrals give a contribution on the order of 1 for altitudes $\leq$ 500 km.) Equating Equations (1.10) and (1.11) yields:

$$\tau_d \sim \left(\frac{L_1^3}{v_0 D_\perp}\right)^{1/2} \sim 3\cdot 10^{-2}. \tag{1.12}$$

It follows from Equations (1.8), (1.10), and (1.12) that when the cloud stops collapsing, it will be a strongly elongated cylinder with dimensions:

$$L_{z_2} \sim v_0 \tau_d \sim 10^5, \qquad L_{y_2} \sim L_{x_2} \sim \left(\frac{L_1^3}{v_0 \tau_d}\right)^{1/2} \sim 10^3 \tag{1.13}$$

2. Motion of a heavy plasma cloud ($\varkappa \gg 1$)

In this case, the cloud dimensions $L_{x_1} \sim L_1$ will be much less than the dimensions $L_{z_1} \sim L_{y_1} \sim v_0 t_1$ when the cloud stops expanding along x [at a time $t \sim t_1$ from Equation (1.5)], when $\beta \sim 1$ and $n \sim n_1 \sim 10^2$ [see (1.6)]. For example, $\varkappa = 137$ for a barium cloud; the numerical values:

$$L_{x_1} \sim v_s t_1 \sim 2\cdot 10^2, \quad L_{z_1} \sim L_{y_1} \sim v_0 t \sim 3\cdot 10^5. \tag{1.14}$$

Since $\beta_y \gg 1$, expansion along y (and also along z) in this case will be such that the cloud becomes a disk, which is flattened along the injection direction (x). Furthermore, the cloud density will continue to decrease, and the magnetic field will collapse the cloud for the same reasons given above, (1), but now only along x. Although compression by the magnetic field is now one-dimensional, nonetheless, the density will again reach the value $n \sim n_1$ ($\beta \sim 1$) after a certain time, since the base surfaces of such a cloud surface grow much faster in time than its lateral surfaces, through which the cloud flows. Actually, the ratio of the cloud particle fluxes perpendicular to these surfaces is of the order: /8

$$\frac{S_{base}}{S_{lateral}} \frac{V_0}{V_A} \sim \frac{(L_{z_1} + v_0 \tau) v_A}{L_x v_0} \sim \left(1 + \frac{\tau}{t_1}\right) \frac{v_A}{v_s} \gg 1 + \frac{\tau}{t_1} \gg 1 \tag{1.15}$$

Thus, the cloud will continue to be compressed along x, and will flow along y and z with an almost constant density $n \sim n_1$. The cloud volume remains constant and deforms as $V \sim x z^2 \sim x y^2$. Then, the same considerations as in section (1) yield:

$$L_z \sim L_y \sim L_{z_1} + v_0\tau, \quad L_x \sim \frac{L_{x_1} L_{z_1}^2}{(L_{z_1}+v_0\tau)^2}, \quad v_x \sim 2\frac{v_0 L_{x_1} L_{z_1}^2}{(L_{z_1}+v_0\tau)^3} \tag{1.16}$$

The diffusion time of the magnetic field is:

$$\tau_d \sim t_1 \sqrt[5]{\frac{L_{x_1}^2}{D_1 t_1}} \sim 3\cdot 10^{-3} \tag{1.17}$$

The cloud dimensions are:

$$L_{x_2} \sim \frac{L_{x_1} L_{z_1}^2}{v_0^2 \tau_d^2} \sim 30, \quad L_{z_2} \sim L_{y_2} \sim v_0 \tau_d \sim 10^4 \tag{1.18}$$

at the moment when the field diffuses completely into the cloud.

It should be noted that a similar discussion of the magnetic /9
field diffusion into the cloud during its collapse is valid under the conditions that the free expansion time t_1 of the cloud is significantly less than the diffusion time τ_d of the magnetic field into the cloud. If instabilities arise in the cloud simultaneously with the collapse of the cloud, and if these instabilities lead to an increase of the effective collision frequency ν_{ef} of the cloud electrons, and thus decrease the (turbulent) diffusion time τ_d, then the cloud collapse will end much more quickly.

c) Stability of the cloud boundary

We consider the stability of a sharp cloud boundary (in the sense that the perturbation wavelength of the boundary is much larger than the thickness of the transition layer) in a coordinate system that is at rest with respect to the undisturbed boundary. Inside the cloud, the electric and magnetic fields are not at equilibrium, and the density is constant. The perturbing quantities are found from the equation of continuity, the equation of state, and the equation of equilibrium in the single-fluid approximation:

$$\begin{gathered} \frac{\partial \rho}{\partial t} + \rho_0 \operatorname{div} \vec{v} = 0, \\ p = \rho_m v_0^2, \\ \rho_0 \frac{\partial \vec{v}}{\partial t} + \nabla p = 0, \end{gathered} \tag{1.19}$$

where $v_S = \sqrt{\gamma_0 p_0/\rho_0}$ is the sound speed; ρ_0 and p_0 are the equilibrium (constant) density and pressure, and γ_0 is the adiabatic index. A perturbation $\vec{v} = \frac{\partial \vec{\xi}}{\partial t}$ from the equilibrium is introduced into the plasma. It is convenient to solve the system of equations (1.19) relative to the variable $\eta = \mathrm{div}\,\vec{\xi}$, instead of for $\vec{\xi}$. If the perturbation is very small scaled, with a wavelength that is small compared with the radius of curvature of the cloud surface, then a local coordinate system can be introduced, and solutions for any perturbed quantity f can be sought in the form: /10

$$f = f(x) \exp(ik_y y + ik_z z - i\omega t) \,, \tag{1.20}$$

where the x-axis is directed perpendicular to the cloud surface, and y and z lie in the plane of the surface. Here, the solution for $\eta(x)$ has the form:

$$\eta(x) = \eta_0 e^{q_i x}, \quad x < x_0 = 0, \tag{1.21}$$

where x_0 is the normal coordinate of the boundary, which can be taken as the origin of the coordinates ($x_0 = 0$); and $q_i = \sqrt{k_y^2 + k_z^2 - \frac{\omega^2}{v_S^2}}$. The displacement $\vec{\xi}$ is expressed in terms of η in the form:

$$\vec{\xi} = \frac{v_S^2}{\omega^2} \nabla \eta . \tag{1.22}$$

A vacuum field exists outside the cloud (we neglect the effect of the surrounding low-density plasma), such that the equilibrium magnetic field is tangent to the surface at the cloud boundary. If the perturbation current is neglected, then the perturbed magnetic field can be written in the form:

$$\vec{B} = \nabla \Psi ; \tag{1.23}$$

Then the equation $\mathrm{div}\,\vec{B} = \Delta \Psi = 0$ yields:

$$\Psi = \Psi_0 e^{-q_e x} \,, \quad x > 0, \tag{1.24}$$

where $q_e = \sqrt{k_y^2 + k_z^2}$.

The dispersion for determining $\omega(\vec{k})$ is obtained by matching the solutions inside and outside the cloud at the boundary. One of the boundary conditions is:

$$\frac{\partial \Psi}{\partial z} + \frac{v_s^2}{\omega^2} B_0 \left(\frac{\omega^2}{v_A^2} \eta - \frac{d \ln B_0}{dx} \frac{\partial \eta}{\partial x} \right) = 0, \tag{1.25}$$

where $v_A = \frac{B_0}{\sqrt{4\pi \rho_0}}$, the Alfven velocity at the cloud boundary, is a result of the constant total (gas-kinetic and magnetic) pressure at the perturbed cloud surface [9]. The second boundary condition is a result of the continuity of the tangential component of the electric field $\vec{E}^* = \vec{E} + [\frac{\vec{v}}{c} \vec{B}]$ at the undisturbed boundary. Here, the Maxwell equation (at the boundary between the two media x = 0), projected on the normal $\vec{n}$ to the boundary, is

$$-\frac{1}{c}\left(\vec{n} \frac{\partial \vec{B}}{\partial t}\right)\Big|_{x>0} = (\vec{n} \operatorname{rot} \vec{E})_{x>0} = (\vec{n} \operatorname{rot} \vec{E}^*)_{x>0} - \left(\vec{n} \operatorname{rot}\left[\frac{\vec{v}}{c} \vec{B}_0\right]\right)\Big|_{x>0} \tag{1.26}$$

Here, we note that the normal component of the curl of a vector can be expressed in terms of the tangential derivatives of the tangential components of the vector. Replacing $\vec{E}^*|_{x>0} = \vec{E}^*|_{x<0}$ (near the cloud x < 0) in Equation (1.26) by its expression from the equation of ion motion*:

$$\vec{E}^* = \frac{m_i}{e_i}\left(\frac{\partial \vec{v}}{\partial t} + \frac{\nabla p_i}{\rho_0}\right) \tag{1.26a}$$

and noting that ρ_0 = const, and $\vec{v}$ has the form $\vec{v} = -i \frac{v_s^2}{\omega} \nabla \eta$ [see (1.22)], yields the following desired boundary condition:

$$\frac{\partial \Psi}{\partial x} - \vec{n} \operatorname{rot}[\vec{\xi} \vec{B}_0] = 0 \tag{1.27}$$

Equations (1.20), (1.21), and (1.24) and the characteristic equations for (1.25) and (1.27) yield the exponential growth rate ($\omega = i\gamma$) for the surface wave**:

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*We do not assume that the electric field lines are trapped $\vec{E}^*|_{x<0} = 0$ [9], since this assumption greatly limits the observed frequency ($\omega \ll \Omega_i$, where $\Omega_i = \frac{e_i B_0}{M_i c}$), although (1.27) is the same as if the field lines are frozen.

**We note that a similar cloud boundary instability was examined in [8].

$$\gamma = v_A \sqrt{\left|k_y \frac{d \ln B_0}{dx}\right|_{x=0} - k_z^2} . \quad (1.28)$$

Here we took $v_A \sim v_s$ from the equilibrium conditions, and the observability conditions ($\lambda \ll \left|-\frac{d \ln B_0}{dx}\right|^{-1}_{x=0} = L$): $k_z^2 < \left|\frac{k_y}{L}\right| \ll k_y^2$ for the existence of an instability, and $\gamma \sim v_A \sqrt{\frac{|k_y|}{L}} \ll k_y v_s$. This flute-like instability (flutes, which extend along the magnetic field $\lambda_z \gg \lambda_y$) is a result of the unfavorable (convex) curvature of the magnetic field containing the cloud ($\frac{d \ln B_0}{dx} < 0$). The perturbation amplitude decreases expotentially on both sides of the cloud boundary on the characteristic line $|\Delta x| = \lambda_x \approx \lambda_y$ (on the order of a wavelength in the y direction). Since we neglected the displacement current, the finite cloud conductivity, the boundary acceleration and the electron inertia, then the expression for the growth rate (1.28) is valid in the region $\tau_d^{-1} \ll \gamma \ll \omega_{pe}, \Omega_e = \frac{eB_0}{mc}$, where $\tau_v = \left|\frac{d \ln v_\perp}{dt}\right|^{-1}$ is the characteristic time for changing the velocity of the observed part of the boundary in a direction perpendicular to the magnetic field. In particular, for the investigated short-wavelength perturbations ($\lambda_y \ll L$), the change in the cloud boundary velocity can always be neglected for times required to develop an instability. The maximum increment can be evaluated by replacing $|k_y^{-1}| \sim \lambda_y$ in Equation (1.28) by its maximum value from the observability condition, which is equal to the transition layer thickness, that is, the penetration depth of the magnetic field onto the cloud $\sim c/\omega_{pe}$. Depending on the cloud collapse and the diffusion of the magnetic field into the cloud, this layer will thicken and the field radius of curvature ∿ L will increase, so that the exponential growth rate will decrease. For an estimate, we introduce the maximum possible value of the growth rate, which is attained at the initial stage of compression of the cloud by the field $(L \sim L_1; |k_y^{-1}| \sim \frac{c}{\omega_{pe}})$:

$$\gamma \sim 10^{-4} \varkappa^{-1/3} . \quad (1.29)$$

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During the time ($\sim \gamma^{-1}$) in which this instability develops, the cloud boundary diffuses to a depth on the order of a perturbation wavelength [10] $\sim \Delta x \simeq \lambda_x \cdot \lambda_y$, so that this instability can lead to an increase of the diffusion of the field into the cloud. As a result, the diffusion time can be less than that found above from a consideration of collisional diffusion, and thus the final transverse cloud dimension can be larger than calculated earlier.

2. Instability of Surface-Potential Oscillations in a Cloud Caused by the Cloud Passing Around the Ionospheric Plasma

The hydromagnetic instability of the cloud boundary, investigated above, was related to the finite unfavorable curvature of the magnetic field surrounding the cloud. Because of the sufficiently rapid cloud expansion along the magnetic field, the curvature of the cloud boundary decreases, so that this instability can be considered not to be dangerous (it does not determine the diffusion of the magnetic field into the cloud). Moreover, it will be shown that the cloud boundary stability is significantly affected by the discharge of the ionospheric plasma, which cannot be considered in the single fluid hydrodynamic model.

Here, we analyze the cloud boundary stability relative to the buildup of potential surface oscillations, caused by the cloud passing around the ionospheric plasma, using a two-fluid hydrodynamic model. The cloud boundary is considered to be a transition region (layer) of thickness $l \ll L$, where L is the cloud dimension. This allows a one-dimensional problem to be solved if the investigated perturbations have a characteristic wavelength $\lambda \ll L$. /14

The ionospheric plasma can be neglected at equilibrium within the transition layer to a first approximation, since the cloud density at this stage of evolution is higher than the density of the ionospheric plasma. If it is assumed for simplicity that the cloud particle temperature is constant, the equilibrium state of the transition layer plasma will be determined by the system of equations:

$$\begin{aligned} -T_\alpha \nabla \ln n_\alpha + e_\alpha\left[E + \frac{v_y}{c}B\right] = 0 \\ -\frac{dB}{dx} = \frac{4\pi}{c}\sum_\alpha e_\alpha n_\alpha v_{y\alpha} \\ \frac{dE}{dx} = 4\pi\sum_\alpha e_\alpha n_\alpha \end{aligned} \qquad (2.1)$$

Here, the index α is either for ions or electrons in the cloud. Equations (2.1) are written in a coordinate system in which the cloud velocity normal to the boundary is zero $(\vec{v} = (0, v_y, v_z))$, the $\vec{x}\,\|\,\vec{\varepsilon}$ axis is normal to the boundary, and the z-axis is along the magnetic field. Considering that the magnetic and electric fields are absent inside the cloud and that only a homogeneous magnetic field exists outside the layer, let us give the electric and magnetic field distribution in the transition layer in the form:

$$E = E_0 f_1(x), \quad B = B_0 f_2(x)$$

such that $f_1(0) = f_2(0) = f_1(\ell) = 0$, $f_2(\ell) = 1$, $E_0 = -\frac{v_{0y}}{c}B_0$, where $v_{0y} = v_y(0)$; $\lim\limits_{x \to 0}\frac{f_1}{f_2} = 1$.

Otherwise, the functions f_1 and f_2 are arbitrary, and should be evaluated from an exact solution of the diffusion of the magnetic field into the plasma. For the given electric and magnetic fields in the layer, the system of equations (2.1) is easily reduced to a system of linear algebraic equations for n_e and n_i: /15

$$\begin{cases} n_e T_e + n_i T_i = n_0 T(1 - f_2^2) \\ n_e - n_i = n_0 \eta f_1' \end{cases} \qquad (2.2)$$

where $n_e(0) = n_i(0) = n_0$, $T = T_e + T_i$, $\eta = \frac{v_{0y} B_0}{4\pi |e| c \ell}$, $f_1' = \ell \frac{\partial f}{\partial x}$.

Equations 2.2 yield:

$$n_\alpha = n_0\left(1 - f_2^2 - \frac{e_\alpha}{|e|}\eta \frac{T_\alpha}{T} f_1'\right), \qquad (2.3)$$

and the first Equation (2.1) yields the quantity:

$$v_{y\alpha} = v_{0y}\left[\frac{f_1}{f_2} - 2\frac{v_{d\alpha}}{v_{0y}}\left(f_2' + \frac{e_\alpha}{|e|}\frac{T_\alpha}{T}\frac{f_1''}{f_2}\right)\right], \qquad (2.4)$$

where $v_{d_\alpha} = \frac{cT}{e_\alpha B_0 l} = \frac{v_T^2}{\Omega_0 l}$, $\Omega_0 = \frac{e_\alpha B_0}{m_\alpha c}$ is the gyro frequency of the α^{th} particle type.

If it is assumed that the boundary layer thickness cannot be less than the quantity c/ω_{pe}, where c is the velocity of light and ω_{pe} is the electron Langmuir frequency, and if the cloud parameters developed above are used, then the quantity η is very small ($\eta \lesssim 10^{-5}$). This means that the plasma is quasi-neutral ($n_e \approx n_i = n_0\left(1 - \frac{B^2}{B_0^2}\right)$) practically everywhere in the transition layer, and that the velocity v_d is less than or of the same order as v_{0y} for the same parameters.

We turn to analyzing the stability of this equilibrium configuration of the surface layer in the presence of a magnetized plasma. Here, a dense nonmagnetized plasma moves along the boundary inside the cloud; a stationary ionospheric plasma is outside the cloud. Cloud and plasma particles are present inside the boundary region. The stability will be investigated in the hydrodynamic approximation. Linearizing the continuity equation and the equation /16 of motion for each particle type, under the assumptions of a perturbing potential ($\vec{E} = -\nabla\varphi$) and unperturbed particle temperatures, yields:

$$n' = -i \operatorname{div} \frac{n\vec{v}^{(0)}}{\omega'}$$
$$\omega'\vec{v}^{(0)} - \frac{1}{\omega'}\left[(\vec{v}^{(0)}\nabla)\vec{v}, \vec{\Omega}\right] + i\left[\vec{v}^{(0)}, \vec{\Omega}\right] = -i\frac{e}{m}\left(\varphi + \frac{T}{e}\frac{n'}{n}\right), \tag{2.5}$$

where $\vec{v}^{(0)} = \vec{v}' + \frac{i}{\omega'}(\vec{v}\nabla)\vec{v}$; $n, \vec{v}'$ are the perturbed density and velocity for particles of a given type; and $\omega' = \omega - \vec{k}\vec{v}$. Since the problem is homogeneous along y and z, all perturbed quantities are proportional to $\exp(ik_y y + ik_z z)$. Combining the Poisson equation:

$$\Delta\Phi = -\sum 4\pi e n', \tag{2.6}$$

with Equation (2.5) yields a closed system of equations which describe the oscillation potential near the cloud boundary. Since the plasma is homogeneous outside the transition layer (to the right and left of the cloud boundary), then the potential $\phi(x)$ can be expressed in the form e^{kx} outside the boundary. Hereafter, we will be interested in only the surface oscillations of the cloud boundary — that is, we will consider that the characteristic wavelength of the perturbations which are perpendicular to the layer to be larger than its thickness: $\kappa \sim \frac{1}{\lambda} \ll \frac{1}{\ell}$. We remember, however, since the problem is one-dimensional, $\lambda \ll L$. Then the potential outside the cloud boundry can be written in the form:

$$\phi(x) = \phi_0 \cdot \begin{cases} e^{\kappa_1 x}, & x < 0 \\ e^{-\kappa_2 x}, & x > 0 \end{cases} \qquad (2.7)$$

Here, we neglect the layer thickness, since $\kappa_2 \ell \ll 1$. The system of Equations (2.5) and (2.6) outside the boundary then reduces to a system of linear homogeneous equations whose solvability condition is the equation: /17

$$K^2 - \sum \frac{\omega_p^2}{\omega'^2} \frac{K^2\omega'^2 - (\vec{K}\vec{\Omega})^2}{\omega'^2 - K^2 v_T^2 - \Omega^2 + v_T^2 \frac{(\vec{K}\vec{\Omega})^2}{\omega'^2}}$$

$$\vec{K} = (K_x, K_y, K_z), \quad K_x = \begin{cases} -i\kappa_1, & x < 0 \\ i\kappa_2, & x > 0 \end{cases} \qquad (2.8)$$

Obviously, Equation (2.8) has a different form on both sides of the boundary. In the cloud region, where $\vec{v} = \vec{v}_0$ this equation relates k_1 with ω' and the cloud parameters, but in the stationary ionospheric plasma region ($\vec{v} = 0$, $\Omega = \Omega_0$) it relates k_2 with ω and with the ionosphere plasma parameters.

The dispersion equation $[D(\omega, k_y, k_z) = 0]$ should be obtained here by considering the boundary conditions. However, in order to write the boundary conditions, first it is necessary to find the concentration n' as a function of the potential ϕ inside the transition region, and then to integrate over the thickness of the layer. The boundary conditions obtained in this manner are insensitive to the gradient of the velocity $\vec{v}$ perpendicular to the boundary. This makes it possible to neglect the difference between the quantities $\vec{v}^{(o)}$ and $\vec{v}^1$ from the very beginning, and to write the solution to the equation of motion [the second of Equations (2.5)] inside the transition layer in the form:

$$\vec{v}^1 = -i\frac{e}{m(\omega'^2-\Omega^2)}\left\{\omega'\nabla\varphi^* - i\left[\nabla\varphi^*, \vec{\Omega}\right] - \frac{\vec{\Omega}}{\omega'}\left(\nabla\varphi^*, \vec{\Omega}\right)\right\} \qquad (2.9)$$

where $\varphi^* = \varphi + \frac{T}{e}\frac{n'}{n}$. Substituting $\vec{v}^1$ in the continuity equation leads to the following equation relating the concentration n^1 with the potential ϕ:

$$\left(\hat{L} - \frac{\omega_p^2}{v_T^2}\right)\frac{n'}{n} = \frac{e}{T}\hat{L}\varphi \qquad (2.10)$$

Here, /18

$$\hat{L} = \frac{\partial}{\partial x}(1-\varepsilon_\perp)\frac{\partial}{\partial x} - k_y^2(1-\varepsilon_\perp) - k_z^2(1-\varepsilon_\parallel) + k_y\frac{\partial g}{\partial x},$$

$$\varepsilon_\perp = 1 - \frac{\omega_p^2(x)}{\omega'^2-\Omega^2(x)}, \quad \varepsilon_\parallel = 1 - \frac{\omega_p^2(x)}{\omega'^2}, \quad g = \frac{\omega_p^2(x)\Omega(x)}{\omega'(\omega'^2-\Omega^2(x))}$$

The explicit dependence of n' on ϕ can be found only in two specific cases: a) for cold particles $(\omega' \gg k v_T)$, when it is possible to neglect the first term in the parentheses of Equation (2.10) on the left side to obtain:

$$n' = -\frac{1}{4\pi e}\hat{L}\varphi \qquad (2.11)$$

and b) hot particles $(\omega' \ll k v_T)$ when the first term is large compared to the second*, and it is possible to write:

$$n' = -\frac{e n_0}{T}\varphi \qquad (2.12)$$

Hereafter, we will assume a cold ionospheric plasma $(\omega \gg k v_{T_{e_2}}, k v_{T_{i_2}}, T_2 \to 0)$, that is, Equation (2.11) will be used for the electrons and ions in the ionospheric plasma, and terms

*Strictly speaking, this hydrodynamic approximation is correct only in these two limiting cases.

proportional to $v_{T_2}^{2}$ will be completely neglected in Equation (2.8). Then, k_z will have the form:

$$k_z = \sqrt{k_y^2 + k_z^2 \frac{(\omega_{pe_2}^2 - \omega^2)(\omega^2 - \Omega_e^2)}{\omega^2 (\omega_{pe_2}^2 + \Omega_e^2 - \omega^2)}} . \tag{2.13}$$

Both the case of cold electrons ($\omega_e' \gg k v_{Te_1}$) and hot electrons ($\omega_e' \ll k v_{Te_1}$) will be considered for the cloud plasma. The cloud ions will be considered to be cold ($\omega_i' \gg k v_{Ti_1}$). /19

a) Cold electron case ($\omega_e' \gg k v_{Te_1}$)

In this case, Equation (2.8) inside the cloud reduces simply to the equation $k^2 = 0$, and yields:

$$k_1 = \sqrt{k_y^2 + k_z^2} = k_\perp . \tag{2.14}$$

Substituting n' from Equation (2.11) for all particles in the Poisson equation (2.6), and integrating over the thickness of the transition layer leads to the following dispersion equation*

$$k_1 \left(1 - \frac{\omega_{pe_1}^2}{\omega'^2}\right) - k_2 \left(1 - \frac{\omega_{pe_2}^2}{\omega^2 - \Omega_e^2}\right) + k_y \frac{\omega_{pe_2}^2 \Omega_e}{\omega(\omega^2 - \Omega_e^2)} = 0 . \tag{2.15}$$

The cloud particle density significantly exceeds that of the ionospheric plasma $n_1 \gg n_2$, that is, $\omega_{pe_1}^2 \gg \omega_{pe_2}^2$. Moreover, the electron gyro-frequency is less than the electron Langmuir frequency ($\Omega_e^2 \ll \omega_{pe_2}^2$) in the ionospheric plasma at altitudes of interest. Then, Equation (2.15) is solved in the frequency region $\omega_e' < \omega_{pe_1}$, $\omega < \omega_{pe_2}$, and is easily determined in two limiting cases $k_y \ll k_z, k_y \gg k_z$.

1) $k_y \gg k_z$. Then $k_1 = k_2 \approx k_y$, and Equation (2.15) has the form:

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*It is easy to see that Equation (2.15) can be obtained directly from the continuity condition at the boundary from the normal component of the electric induction.

$$\frac{\omega_{pe_2}^2}{\omega^2 - \Omega_0^2} + \frac{\omega_{pe_1}^2}{\omega'^2} - \frac{\omega_{pe_2}^2 \Omega_0}{\omega(\omega^2 - \Omega_0^2)} = 0. \tag{2.16}$$

Since $\omega_{pe_1}^2 \gg \omega_{pe_2}^2$, then $\omega' \simeq -\vec{k}\,\vec{v}_{0e}$, and

$$\omega \simeq -\frac{1}{2}\Omega_0 \left[1 \pm \sqrt{1 - 4\frac{n_2}{n_1}\frac{(\vec{k}\,\vec{v}_0)^2}{\Omega_0^2}}\right]. \tag{2.17}$$

Equation (2.17) shows that oscillations with /20

$$\left|\frac{k_y v_{0y}}{\Omega}\right| > \frac{1}{2}\sqrt{\frac{n_1}{n_2}} \tag{2.18}$$

are unstable with an increment

$$\gamma \simeq |k_y v_{0y}|\sqrt{\frac{n_2}{n_1}} > \Omega_0. \tag{2.19}$$

The electric fields which arise in this instability are directed perpendicular to the magnetic field and can significantly affect the current distribution in the layer, which in turn significantly changes the diffusion of the magnetic field into the cloud. For the case:

2) $k_y \ll k_z$, $k_1 \simeq |k_z|$, $k_2 \simeq |k_z|\sqrt{\frac{\omega^2 - \Omega_0^2}{\omega^2}}$

Equation (2.15) reduces to the form

$$\frac{\omega_{pe_2}^2}{\omega(\omega^2 - \Omega_0^2)^{1/2}} + \frac{\omega_{pe_1}^2}{k_z^2 v_{0z}^2} = 0 \tag{2.20}$$

and has an unstable solution $\omega = i\gamma$, where

$$\gamma \simeq |k_z v_{0z}|\sqrt{\frac{n_2}{n_1}} > |\Omega_0|. \tag{2.21}$$

An examination of these limiting cases makes it obvious that for $|\vec{k}\vec{v}_0| > \Omega_0$, an instability develops for which the thermal velocity dispersion of the cloud can be neglected. However, if we take $k_1 \approx k_\perp$ for these instabilities, then it turns out that instabilities with $k_1 > \Omega_0/|\vec{v}_0|$ occur only for bundle velocities significantly above the thermal velocity of the cloud electrons. Actually, it follows from Equation (2.19) [see also (2.[illegible])] that:

$$|\vec{v}_0| > \frac{|\Omega_0|}{k_\perp}\left(\frac{n_1}{n_2}\right)^{1/2} > |\Omega_0|\ell\left(\frac{n_1}{n_2}\right)^{1/2} > v_{Te}\frac{\ell}{\rho_0}\left(\frac{n_1}{n_2}\right)^{1/2} \gtrsim v_{Te}. \qquad (2.22)$$

Since we obtain $|\vec{v}_0| \lesssim v_{Te}$ experimentally, then the hot electron case is of greater interest. /21

b) Hot electron case ($\omega_e' \ll k v_{Te}$)

Here, the solution of Equation (2.8) for k_1, that is, the projection of the wave vector on the x-axis inside the cloud has the form:

$$k_1 = \left(k_\perp^2 - \frac{\omega'^2}{v_s^2}\right)^{1/2}, \qquad (2.23)$$

where $v_s^2 = T_{e_1}/M_i$, and M_i is the mass of cloud ions. The concentration n' is now related to the potential ϕ for cloud electrons by Equation (2.12). Then, integrating the Poisson equation over the thickness of the transition layer leads to the following dispersion equation:

$$k_1\left(1 - \frac{\omega_{pi_1}^2}{\omega'^2}\right) + k_2\left(1 - \frac{\omega_{pe_2}^2}{\omega^2 - \Omega_0^2}\right) + k_y\frac{\omega_{pe_2}^2\Omega_0}{\omega(\omega^2 - \Omega_0^2)} = 0, \qquad (2.24)$$

where k_1 is determined from Equation (2.23), and k_2 is determined only by assuming an ionospheric plasma which is not too rarefied $\left(\frac{m}{M_i} \ll \frac{n_2}{n_1} \ll 1\right)$. Then, in the first approximation $\omega \simeq \vec{k}\vec{v}_0$, $\omega' = \omega - \vec{k}\vec{v}_0 = i\gamma$, and Equation (2.24) takes the form:

$$\left(1 + \frac{\omega_{pi_1}^2}{\gamma^2}\right)\left(1 + \frac{\gamma^2}{k_\perp^2 v_s^2}\right)^{1/2} = q(\vec{k}\vec{v}_0), \qquad (2.25)$$

after substituting k_1 and k_2, where:

$$q(\omega) = \frac{k_2(\omega)}{k_\perp}\left(1 - \frac{\omega_{pe_2}^2}{\omega^2 - \Omega_0^2}\right) + \frac{k_y}{k_\perp}\frac{\omega_{pe_2}^2\Omega_0}{\omega(\omega^2 - \Omega_0)}. \qquad (2.26)$$

If the left side of Equation (2.25) is taken as a function of $\gamma^2 > 0$, then it can be seen that it has a minimum value $q/k_\perp r_{De}$, where $r_{De} = v_{Te}/\omega_{pe_1}$ is the Debye radius of the cloud electrons, under the condition that $k_\perp r_{De} \ll 1$ ($r_{De} < \ell < k_y^{-1} \sim k_\perp^{-1}$). From this it follows that a /22 purely aperiodic solution ($\gamma^2>0$) occurs for large values of $q(\vec{k}\vec{v}_0)$. Studying the behavior of q as a function of $\vec{k}\vec{v}_0$, we note that q is large when $(\vec{k}\vec{v}_0)^2 \simeq \Omega_0^2$. Introducing the notation $y = \vec{k}\vec{v}_0/\Omega_0$ in the circle $y^2 \approx 1$ yields:

$$q \simeq \frac{K_y}{K_\perp} \frac{\omega_{p_{e_2}}^2}{\Omega_0^2} \frac{1}{y + \operatorname{sign} K_y} \qquad (2.27)$$

It is obvious that $q>0$ for $|y|<1$, that is, the instability occurs for $|\vec{k}\vec{v}_0|<|\Omega_0|$. On the other hand, $\gamma^2>0$ requires that $q > 2/K_\perp r_{D_e}$, that is, the condition for the existence of an aperiodic instability in the system of a stationary cloud has a resonant character:

$$0 < 1 - \left|\frac{\vec{K}\vec{v}_0}{\Omega_0}\right| < \frac{1}{2} |K_y| r_{D_{e_1}} \frac{\omega_{p_{e_2}}^2}{\Omega_0^2}. \qquad (2.28)$$

The exponential growth rate of this instability is:

$$\gamma \simeq K_\perp v_s q \qquad (2.29)$$

and is larger than the cloud Langmuir frequency ($\gamma > \omega_{p_{i_1}}$.).

A drift instability can exist along with the highly resonant aperiodic instability, if $1 < q < 2/r_{D_e} K_\perp$. Here, if $q \gg 1$, then the solution to Equation (2.25) is found in the region $|\gamma| \gg K_\perp v_s$, and has the form:

$$\gamma = \frac{1}{2} q K_\perp v_s \pm i\omega_{i_1}^2 \left(1 - \frac{1}{4} q^2 K_\perp^2 r_{D_e}^2\right)^{1/2}. \qquad (2.30)$$

Thus, outside the resonance, determined by (2.28), there are two branches of unstable oscillations with frequencies:

$$\omega = \vec{k}\vec{v}_0 \pm \omega_r \left(1 - \frac{1}{4} q^2 K_\perp^2 r_{D_e}^2\right)^{1/2} \qquad (2.31)$$

and with an exponential growth rate of: /23

$$\operatorname{Im} \omega = \frac{1}{2} q K_\perp v_s, \qquad (2.32)$$

which, however, is less than the Langmuir frequency of the cloud ($\operatorname{Im} \omega < \omega_{p_{i_1}}$).

In conclusion, it should be noted that since the directed cloud velocity $|\vec{v}_0| \lesssim v_{Te}$ and, under the conditions being investigated, $\kappa_\parallel \sim \kappa_\perp \ll \frac{1}{\ell} < \frac{\Omega_0}{v_{Te}} < \frac{\Omega_0}{|\vec{v}_0|}$, the Inequality (2.28) can be satisfied only for $(\Omega_0/\omega_{Pe_2})(\frac{n_1}{n_2})^{1/2} < 1$. This condition is not satisfied for experiments of interest to us. Thus, the maximum exponential growth rate for the surface potential oscillation instability of the cloud, when passing over the ionosphere plasma, is less than the Langmuir frequency of the cloud ions. However, in this case it significantly exceeds the growth rate for hydromagnetic instability. Thus, this instability can play an important role in the diffusion of a magnetic field into the cloud plasma.

3. Stability of Weakly Inhomogeneous Anisotropic Plasma with a Finite Pressure

(as applied to a cloud at its diffusion expansion stage)

a) Initial equations

The hydrodynamic instability of the boundary and the instability of surface oscillations of the cloud when passing over the ionosphere plasma are related to the end of the first stage — the free expansion stage of the cloud — and to the start of the second stage — the stage of diffusion of the magnetic field into the cloud. These instabilities basically determine the diffusion velocity of the magnetic field into the plasma, but they cannot qualitatively change the character of the cloud evolution, since they only occur in the presence of a thin transition layer between the magnetic field and the cloud plasma. Depending on the penetration of the magnetic field into the plasma, the transition layer thickness increases, and becomes equal with the cloud dimensions. In this case, volume perturbations start to play an important role. These perturbations have a wavelength which is less than the dimensions of the cloud inhomogeneities and of the magnetic field. We use a kinetic approach to analyze these perturbations. This approach allows us to consider the effect of pressure (temperature) anisotropy in the cloud plasma, which arises from the free flow of the /24 plasma along the magnetic field ($T_\perp \gtrsim T_\parallel$). The effect of the surrounding ionospheric plasma will be neglected for the time being (see below). The problem can be considered to be one-dimensional in the case of a small-scale perturbation ($\kappa L \gg 1$), which will interest us further on*. Here it is not necessary to use the coordinate system with the x-axis along the direction of the inhomogeneity. It is more convenient to choose a coordinate system such that $\vec{\kappa}_\perp$ (the oscillation wave vector projected on a plane perpendicular to the magnetic field) is perpendicular to the x-axis. Then the projection of the wave vector along the inhomogeneity is $\kappa_1 = \kappa_\perp \cos\alpha$, and $\kappa_2 = \kappa_\perp \sin\alpha$ perpendicular to the inhomogeniety, where the angle α is the angle between the direction of the inhomogeneity and the vector $\vec{K}_\perp$.

The dispersion equation for small oscillations in the quasi-classical approximation [the zero approximation with respect to the parameter $(k_\perp L)^{-1}$] is an algebraic equation, and has the known form:

$$\det\left(N^2\delta_{ij} - N_i N_j - \varepsilon_{ij}\right) = 0, \tag{3.1}$$

where $\vec{N} = \frac{\vec{k}c}{\omega}$ is the refraction index of the medium, and /25

$\varepsilon_{ij} = \delta_{ij} + \sum_\alpha \left(\varepsilon_{ij}^{(\alpha)} - \delta_{ij}\right)$ is the dielectric constant tensor of the medium ($\varepsilon_{ij}^{(\alpha)}$ is the dielectric constant tensor of the α^{th} type.particles). If we introduce the angle Θ between the wave vector $\vec{k}$ and the z-axis, which is directed along the magnetic field ($\operatorname{tg}\Theta = k_\perp/k_z$), then, in the chosen coordinate system with $k_x = 0$, Equation (3.1) takes the form:

$$\begin{aligned}
&(N^2 - \varepsilon_{xx})\left\{\varepsilon_{yy}\varepsilon_{zz} - \varepsilon_{yz}\varepsilon_{zy} - N^2\left[\varepsilon_{yy}\sin^2\Theta + \varepsilon_{zz}\cos^2\Theta + \right.\right. \\
&\left.\left. + (\varepsilon_{yz} + \varepsilon_{zy})\sin\Theta\cos\Theta\right]\right\} - \varepsilon_{xy}\varepsilon_{yx}\left(N^2\sin^2\Theta - \varepsilon_{zz}\right) - \\
&- \varepsilon_{xz}\varepsilon_{zx}\left(N^2\cos^2\Theta - \varepsilon_{yy}\right) - \varepsilon_{xy}\varepsilon_{zx}\left(N^2\sin\Theta\cos\Theta + \varepsilon_{yz}\right) \\
&- \varepsilon_{yx}\varepsilon_{xz}\left(N^2\sin\Theta\cos\Theta + \varepsilon_{zy}\right) = 0.
\end{aligned} \tag{3.2}$$

*We will also neglect the curvature of the magnetic field lines, which is correct in this stage, when the cloud is mainly stretched along the magnetic field (here, the field lines are only weakly distorted).

The electron dielectron constant for the cloud can be determined using the approximation of a weak inhomogeneity, since the electron Larmor radius is small compared to the characteristic dimension of the inhomogeniety. The state of the ions in the diffusional cloud expansion passes through two stages. In the first stage, the Larmor radius of the ions is large compared to the cloud dimensions. In this case, the calculation of the ionic contribution to the dielectric constant can neglect the effect of the magnetic field (the case of unmagnetized ions). As the cloud expands further, its dimensions increase and become larger than the ionic Larmor radius. Then we have the second stage — the stage of magnetized ions when the approximation of a weak inhomogeneity is applicable to the ions as well as for the electrons. Thus, the dielectric constant tensor for a given particle type is determined in two limiting cases: $\rho \gg L$ and $\rho \ll 1$, where $\rho = v_{T\perp}/|\Omega|$ is the Larmor radius of the particles. /26

1. $\rho \gg L$

In this case, as noted earlier, the effect of the external magnetic field on the motion of a given particle type can be neglected, and the binormal distribution function:

$$f_0 = F_0 = \frac{n(\vec{r})\, m^{3/2}}{(2\pi)^{3/2} T_\perp(\vec{r})\, T_\parallel^{1/2}(\vec{r})} \exp\left[-\frac{m}{2}\left(\frac{v_\perp^2}{T_\perp(\vec{r})} + \frac{v_\parallel^2}{T_\parallel(\vec{r})}\right)\right] \qquad (3.3)$$

can be used as the equilibrium distribution function. The perturbation contribution to the distribution function is determined from the known linearized Vlasov equation (we neglect collisions between particles), and has the form:

$$f = \frac{ie}{m\omega(\omega - \vec{k}\vec{v})}\left(\frac{\vec{v}_\perp \vec{E}}{v_\perp}\Phi_1 + E_z \Phi_2\right) F_0, \qquad (3.4)$$

where

$$\Phi_1 = \frac{v_\perp}{v_{T\perp}^2}\left(\omega + k_z v_\parallel \frac{\Delta T}{T_\parallel}\right),$$

$$\Phi_2 = \frac{v_\parallel}{v_{T\parallel}^2}\left(\omega - \vec{k}_\perp v_\perp \frac{\Delta T}{T_\perp}\right),$$

$$\Delta T = T_\perp - T_\parallel.$$

The dielectric constant tensor is related to the conductivity tensor by the relation $\varepsilon_{ij} = \delta_{ij} + \frac{4\pi i}{\omega}\sigma_{ij}$, which is determined from the equation:

$$j_i = e\int v_i f d\vec{v} = \sigma_{ij} E_j ; \tag{3.5}$$

where $\vec{j}$ is the perturbed flux of a given particle type. Integration over velocity in Equation (3.5) in the general case is complicated. So we consider two limiting cases: 1) wavespropagated perpendicular to the magnetic field ($\omega > k_z v_{T_\parallel}$), and 2) waves propagated along the magnetic field ($\omega \gg k_\perp v_{T\perp}$). It is obvious that in the limit of cold particles ($\omega \gg k v_{T_{\perp,\parallel}}$), both cases should give the same result. In the /27 first case, the multiplier $(\omega - \vec{k}\vec{v})^{-1}$ in Equation (3.4) for f is expanded in powers of $k_z v_\parallel (\omega - \vec{k}_\perp \vec{v}_\perp)^{-1}$. In the second case, it is expanded in power of $\vec{k}_\perp \vec{v}_\perp (\omega - k_z v_\parallel)^{-1}$. After integrating Equation (3.5) over velocity, this procedure yields the following expression for the dielectric constant tensor:

$$\varepsilon_{ij} - \frac{\omega_p^2}{\omega^2}\Lambda_{ij} , \tag{3.6}$$

where, in the case $\omega \gg k_z v_{T_\parallel}$, the tensor Λ_{ij} has the components:

$$\begin{gathered} \Lambda_{xx} = J_+(x_\perp) , \quad \Lambda_{yy} = x_\perp^2 [J_+(x_\perp) - 1] \\ \Lambda_{zz} = J_+(x_\perp) - \frac{\Delta T}{T} [J_+(x_\perp) - 1] \\ \Lambda_{yz} = \Lambda_{zy} = -\frac{k_z}{k_\perp}\left[J_+(x_\perp) + x_\perp^2\left(\frac{J_+}{x_\perp}\right)' + \frac{\Delta T}{T_\perp}\left(1 - 2J_+(x_\perp) - x_\perp^2\left(\frac{J_+}{x_\perp}\right)\right)\right] \end{gathered} \tag{3.7}$$

The components in the second case $\omega \gg k_\perp v_{T\perp}$:

$$\begin{gathered} \Lambda_{xx} = \Lambda_{yy} = J_+(x_\parallel) + \frac{\Delta T}{T_\parallel}\left(J_+(x_\parallel) - 1\right), \quad \Lambda_{zz} = x_\parallel^2 \left(J_+(x_\parallel) - 1\right) \\ \Lambda_{yz} = \Lambda_{zy} = -\frac{k_\perp}{k_z}\left[J_+(x_\parallel) + x_\parallel^2\left(\frac{J_+(x_\parallel)}{x_\parallel}\right)' - \frac{\Delta T}{T}\left(1 - 2J_+(x_\parallel) + x_\parallel^2\left(\frac{J_+(x_\parallel)}{x_\parallel}\right)\right)\right]. \end{gathered} \tag{3.8}$$

Here, $J_+(\beta) = \frac{\beta}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\frac{dx}{\beta - x} e^{-\frac{x^2}{2}}$, $x_{\perp,\parallel} = \frac{\omega}{|k_{\perp,z}| v_{T_{\perp,\parallel}}}$. The other components of Λ_{ij} are equal to zero. It is easy to see that the Expressions (3.7) and (3.8) for Λ_{ij} coincide in the limit $x_{\perp,\parallel} \gg 1$, when $J_+(x_{\perp,\parallel}) \simeq 1 + x_{\perp,\parallel}^{-2}$.

II. $\rho \ll L$

In this case, we examine the dynamics of the cloud electrons and ions in the second expansion stage. In the first approximation of a weak inhomogeniety, the distribution function for a given particle type is chosen as close to a binormal function: /28

$$f_0(\vec{r},\vec{v}) \approx \left(1 - \frac{\vec{\Omega}[\vec{v}\,\nabla]}{\Omega^2}\right) F_0(\vec{r},\vec{v}), \tag{3.9}$$

where $f_0(\vec{r},\vec{v})$ is determined by Equation (3.3), and $\vec{\Omega} = \frac{e\vec{B}}{mc}$. The perturbation distribution function is written in the form of a time integral along the unperturbed trajectory of particle motion:

$$f = -\frac{e}{m\omega}\int_{-\infty}^{0} dt'\left[(\omega - \vec{k}\vec{v}')\vec{E} + \vec{k}(\vec{v}'\vec{E})\right]\frac{\partial f_0}{\partial \vec{v}'}\exp\left[-i\omega t' + i(\vec{k}\Delta\vec{r})\right] \tag{3.10}$$

Here is it convenient to write the particle velocity as a function of t' in projections on the direction of inhomogeneity (v_1'), and the direction perpendicular to the inhomogeneity (v_2') (the direction of particle drift):

$$v_1' = v_\perp \cos(\vartheta - \Omega t'),\quad v_2' = v_\perp \sin(\vartheta - \Omega t') + \frac{v_\perp^2}{2\Omega}\frac{d\ln\Omega}{dx}$$
$$v_3' = v_z' = v_z \tag{3.11}$$

The value of $\vec{k}\Delta\vec{r}$ along the particle trajectory has the form:

$$\vec{k}\Delta\vec{r} = k_z v_\parallel t' + \frac{k_\perp v_\perp}{\Omega}\left[\cos(\vartheta - \alpha - \Omega t') - \cos(\vartheta - \alpha)\right] + \frac{k_2 v_\perp^2}{2\Omega}\frac{d\ln\Omega}{dx}t' \tag{3.12}$$

The angle ϑ is the azimuthal angle in velocity space between $v_\perp$ and the direction of the inhomogeneity. Substituting Equation (3.10) for the perturbed distribution function in (3.5), and integrating over t', ϑ and $v_\parallel$ determines σ_{ij} and also ε_{ij} only in terms of an integral for $v_\perp$. Here, the dielectric constant tensor ε_{ij} has the form:

$$\varepsilon_{ij} = \delta_{ij} - \frac{\omega_p^2}{\omega^2}\sum_{n=-\infty}^{\infty}\frac{1}{v_{T_\perp}^2}\langle \Lambda_j \Lambda_i^* \rangle,\quad i \neq z,$$

$$\varepsilon_{zj} = \delta_{zj} - \frac{\omega_p^2}{\omega^2}\sum_{n=-\infty}^{\infty}\frac{1}{v_{T_\perp}^2}\left\langle \Lambda_j \frac{\bar{\omega} - n\Omega}{k_z} J_n \right\rangle \tag{3.13}$$

Here, the vector $\vec{\Lambda}$ has the projections: /29

$$
\begin{aligned}
\Lambda_x &= \left[\hat{\ell}_\perp \frac{\omega}{\bar{\omega} - n\Omega} J_t + \frac{\Delta T}{T_\parallel}(J_t - 1)\right] \overset{*}{A}_x \\
\Lambda_y &= \left[\hat{\ell}_\perp \frac{\omega}{\bar{\omega} - n\Omega} J_t + \frac{\Delta T}{T_\parallel}(J_t - 1)\right] A_y \\
\Lambda_z &= \left[\hat{\ell}_\perp + \frac{\bar{\omega} - n\Omega}{\omega} \frac{\Delta T}{T_\parallel}\right](J_t - 1) \frac{\omega}{k_z} J_n
\end{aligned}
\tag{3.14}
$$

Also, $\overset{*}{A}_x$ and $\overset{*}{A}_y$ are the complex conjugates of the quantities A_x and A_y:

$$
A_x = -i v_\perp J_n' \, ; \quad A_y = n \frac{\Omega}{k_\perp} J_n \, .
$$

The function J_n is the Bessel function of the argument $k_\perp v_\perp / \Omega$. The function J_t is determined the same way as before [see Equation (3.8)], and only depends on the argument $\bar{\omega} - n\Omega / |k_z| v_{T\parallel}$. The other notations have the form:

$$
\hat{\ell}_\perp = 1 - \frac{k_z v_{T\perp}^2}{\Omega \omega L}, \quad L^{-1} = \frac{1}{k_\perp}(\vec{k}\nabla); \quad \bar{\omega} = \omega - \frac{k_z v_{T\perp}^2}{\Omega k_\perp}(\vec{k}\nabla \ln n) . \tag{3.15}
$$

The brackets $\langle \ldots \rangle$ indicate integration over $v_\perp$ with a weight:

$$
\frac{1}{2\pi} \frac{v_\perp}{v_{T\perp}^2} \exp\left(-\frac{v_\perp^2}{2 v_{T\perp}^2}\right)
$$

b) Electron oscillations in a cloud with unmagnetized ions (ρ_i >> L)

As we mentioned earlier, when the magnetic field has almost completely diffused into the cloud, the transverse dimension of the cloud becomes small compared to the ion Larmor radius. At this stage of the cloud expansion, the ions can be considered to be unmagnetized. They can be described by the dielectric constant found in section 1. The electron component is always magnetized (even in the plasma transition layer). Thus, its dielectric properties are described by the dielectric constant tensor for section 2 at any stage of expansion.

This paragraph treats oscillations with frequencies /30 $\omega \gtrsim \omega_d \approx \frac{k_z v_{Ti}^2}{\Omega L} \sim k_z v_{Ti} \frac{\rho_i}{L} \gg k_z v_{Ti}$. For such oscillations, the ions are cold and "infinitely heavy", and make only a small contribution to the dielectric constant. We restrict ourselves to studying long-wave ($\kappa_\perp \rho_e \ll 1$), low-frequency ($\omega \lesssim \kappa_\parallel v_{T_\parallel} < \Omega_e$) perturbations, which are sensitive to relatively small temperature anisotropies. Then the components of the dielectric constant tensor for the electrons (and thus for the whole plasma) can be written in the form:

$$\varepsilon_{ij} = \varepsilon_{ij}^{(3)} + \varepsilon_{ij}^{(a)}, \tag{3.16}$$

where $\varepsilon_{ij}^{(3)}$ is the Hermitian part of the dielectric constant tensor, which characterizes the hydrodynamically unstable spectra of the oscillations:

$$\begin{aligned}
&\varepsilon_{xx}^{(3)} = N^2\left[\frac{\hat{\omega}_d' \omega_m'}{\kappa_\perp^2 v_{T_\perp}^2} \cdot \frac{\beta_\perp}{2} + \frac{\Delta T}{T_\parallel}\left(\beta_\perp - \frac{1}{2}\beta_\parallel \operatorname{ctg}^2\Theta\right)\right]\sin^2\Theta \\
&\varepsilon_{yy}^{(3)} = N^2\left[\frac{\hat{\omega}_d' \omega_m'}{\kappa_\perp^2 v_{T_\perp}^2} \cdot \frac{\beta_\perp}{2} - \frac{\beta_\parallel}{2}\frac{\Delta T}{T_\parallel}\right]\cos^2\Theta \\
&\varepsilon_{zz}^{(3)} = N^2\left[\frac{\omega_p^2}{\kappa_\perp^2 c^2}\frac{\hat{\omega}_d' \omega_m''}{\kappa_\perp^2 v_{T_\perp}^2}\left(1 - \frac{\langle\bar{\omega}^2\rangle}{\omega_d' \omega_m''}\frac{\Delta T}{T_\parallel}\right) - \frac{\beta_\parallel}{2}\frac{\Delta T}{T_\parallel}\right]\sin^2\Theta \\
&\varepsilon_{xy}^{(3)} = -\varepsilon_{yx}^{(3)} = i\,\frac{\omega_p^2 \hat{\omega}_d'}{\omega^2 \Omega}; \quad \varepsilon_{xz}^{(3)} = -\varepsilon_{zx}^{(3)} = -i\,\frac{\omega_p^2 \hat{\omega}_d'}{\omega^2 \Omega}\left(1 + \frac{\omega_m'}{\omega_d'}\frac{\Delta T}{T_\parallel}\right)\operatorname{tg}\Theta \\
&\varepsilon_{yz}^{(3)} = \varepsilon_{zy}^{(3)} = N^2\frac{\beta_\parallel}{2}\frac{\Delta T}{T_\parallel}\sin\Theta\cos\Theta.
\end{aligned} \tag{3.17}$$

Here, $\beta_{\perp,\parallel} = \frac{8\pi n T_{\perp,\parallel}}{B^2}$ is the ratio of the gas-dynamic pressure of the plasma to the magnetic pressure, $\hat{\omega}_d' = \omega - \hat{\omega}_d$, $\hat{\omega}_d = \frac{\kappa_2 v_{T_\perp}^2}{\Omega}\hat{L}^{-1}$, and $\omega_m = \frac{1}{2}\left\langle \frac{v_\perp^2}{v_{T_\perp}^2}\bar{\omega}\right\rangle$, $\omega_m' \langle\bar{\omega}\rangle$.

Expressions for $\varepsilon_{ij}^{(a)}$, the anti-Hermitian part of the dielectric constant tensor, will be given later, after analysis of the hydrodynamic stability. Substituting ε_{ij} in Equation (3.2), and completely neglecting the anti-Hermitian part $\varepsilon_{ij}^{(0)}$, yields the following equation (for simplicity, we take T = const. Then $\hat{\omega}_d$ reduces to $\omega_d = \frac{k_2 v_{T_\perp}^2}{\Omega}\frac{d\ln n}{dx}$): /31

$$\frac{\omega_d'^2}{\Omega^2}\frac{\omega_p^4}{k^2 k_z^2 c^4} = \left(1 + \frac{\beta_\parallel}{2}\frac{\Delta T}{T_\parallel}\right)\left\{1 - \frac{\Delta T}{T_\parallel}\left[\beta_\perp - \frac{\beta_\parallel}{2}\operatorname{ctg}^2\Theta - \frac{\beta_\perp}{2}\frac{\omega_m'^2}{\omega_d'^2}\frac{\Delta T}{T_\parallel}\left(1 + \frac{\langle\bar{\omega}^2\rangle}{\omega_d' \omega_m''}\frac{\Delta T}{T_\parallel}\right)^{-1}\right]\cdot\sin^2\Theta\right\}. \tag{3.18}$$

Equation (3.18) describes two independent oscillation modes for a plasma whose pressure is not too high ($\beta < kL$). One of these modes lies in the frequency region $\omega \gg \omega_{d,H}$, is not related to the plasma inhomogeneity, and is called whistler (or whistler atmospherics). The second is intrinsically related to the presence of anisotropy and inhomogeneity, that is, it is the "drift-anisotropic" mode.

The oscillation spectrum for the whistler mode has the form:

$$\omega = \pm\omega_0 = \pm\Omega \frac{k^2c^2}{\omega_p^2}\left(1+\frac{\beta_\perp}{2}\frac{\Delta T}{T_\parallel}\right)^{1/2}\left[1-\frac{\Delta T}{T_\parallel}\sin^2\theta\Big/\left(\frac{2}{\beta_\parallel}-\frac{\beta_\parallel}{2}\mathrm{ctg}^2\theta-\frac{\beta_\perp}{2}\frac{\Delta T}{T_\parallel}\right)\right]^{1/2}\cos\theta \quad (3.19)$$

and is purely imaginary, that is, the oscillations are aperiodically unstable if the inequality

$$\chi\sin^2\theta > 1, \quad (3.20)$$

is satisfied, where $\chi = \frac{\Delta(3+\Delta)}{\Delta+2/\beta_\parallel}$, $\Delta = \Delta T/T_\parallel$. Analysis of Inequality (3.20) shows that stabilities only occur in regions of positive ΔT when:

$$\frac{\Delta T}{T_\parallel} > \Delta_+ = -1+\sqrt{1+2/\beta_\parallel}, \quad (3.21)$$

where Δ_+ is the first root of the equation $\chi(\Delta)=1$. The correspond- /32
ing interval for the quantity $\Delta T/T_\parallel\ (\Delta T<0)$ has the form:

$$\frac{\Delta T}{T_\parallel} < -\frac{2}{\beta_\parallel}, \ \text{т.е. } \beta_\parallel > 2. \quad (3.23)$$

We note that in the hydrodynamic whistler instability, oscillations are excited with a large ratio $\frac{K_\perp}{K_z}(\sin^2\theta \to 1)$, so that there is always a minimum value $\frac{K_\perp^2}{K^2} \approx \sin^2\theta$, below which the oscillations are hydrodynamically stable (see section 4).

If the conditions for a hydrodynamic instability are not satisfied in the system, then it is necessary to study the possibility of the existence of a kinetic instability. Then the quantity $K_\perp$ is not very small, and the fundamental contribution in the anti-Hermitian part of the dielectric constant tensor comes from terms proportional to $J_+\left(\frac{\omega}{|k_\parallel| v_{T_\parallel}}\right) \approx \frac{\omega}{|k_\parallel| v_{T_\parallel}} \sqrt{\frac{\pi}{2}}$. In this case, the tensor components $\varepsilon_{ij}^{(a)}$ will have the form:

$$\varepsilon_{xx}^{(a)} = -2\frac{\omega_p^2}{\omega^2} k_\perp^2 \rho^2 \frac{T_\perp}{T_\parallel} J_+ , \quad \varepsilon_{yy}^{(a)} = \varepsilon_{xy}^{(a)} = \varepsilon_{yx}^{(a)} = \varepsilon_{yz}^{(a)} = \varepsilon_{zy}^{(a)} = 0,$$
$$\varepsilon_{xz}^{(a)} = -\varepsilon_{zx}^{(a)} = i\frac{\omega_p^2}{\omega \Omega} \frac{k_\perp}{k_\parallel} \frac{T_\perp}{T_\parallel} J_+ , \quad \varepsilon_{zz}^{(a)} = -\frac{\omega \rho^2}{k_\parallel^2 v_{T_\parallel}^2} J_+ , \tag{3.24}$$

and the dispersion equation (3.2) reduces to the equation:

$$D^{(2)} + D^{(a)} J_+ = 0,$$
$$D^{(2)} = \frac{\omega_p^6}{\omega^3 \Omega^2 k_\parallel^2 v_{T_\parallel}^2} \left(1 - \frac{\omega_0^2}{\omega^2}\right)$$
$$D^{(a)} = -N^4 \frac{\omega_p^2}{k^2 v_{T_\parallel}^2} \left(1 + \frac{(\Delta T)^2}{2 T_\perp T_\parallel}\right)\left(1 + \frac{\beta_\parallel}{2} \frac{\Delta T}{T_\parallel}\right) \sin^2 \vartheta . \tag{3.25}$$

The solution to Equation (3.25) has the form $\omega = \pm\omega_0 + i\gamma$, where the increment γ is: /33

$$\gamma = -\frac{1}{2} \sqrt{\frac{\pi}{2}} \frac{\Omega^2 k^4 c^4}{|k_\parallel| v_{T_\parallel} \omega_p^4} \left(1 + \frac{(\Delta T)^2}{T_\perp T_\parallel}\right)\left(1 + \frac{\beta_\parallel}{2} \frac{\Delta T}{T_\parallel}\right) \sin^2\vartheta \cos^2\vartheta \tag{3.26}$$

and is positive only for negative Δi:

$$\frac{\Delta T}{T_\parallel} < -\frac{2}{\beta_\parallel} \tag{3.27}$$

Condition (3.27) shows that the kinetic instability, which arises from ordinary particle-interaction resonances with the wave ($\omega = k_\parallel v_\parallel$) is possible for the same degree of anisotropy as the hydrodynamic instability. If the degree of anisotropy is small ($\frac{|\Delta T|}{T_\parallel} < 2/\beta_\parallel$), then studying the stability of whistlers requires an examination of oscillations with a sufficiently small ratio $k_\perp/k_z$, when the resonant reaction of particles with the wave starts to play an important role at the cyclotron frequency ($\omega \mp \Omega \approx k_\parallel v_\parallel$). Examination of such oscillations is simplified by the fact that the fundamental terms in the dispersion equation are proportional to ε_{zz}. Taking ε_{zz} different from zero and using $\cos^2\vartheta \approx 1$ in the first approximation yields the following equation from (3.2):

$$(N^2 - \varepsilon_{xx})(N^2 - \varepsilon_{yy}) - \varepsilon_{xy}\varepsilon_{yx} = 0, \tag{3.28}$$

which, in the frequency and wavelength region of interest to us, transforms to the form ($\frac{\Delta T}{T_\parallel} < 1/\beta_0$):

$$\left(1+\frac{\beta_0}{2}\frac{\Delta T}{T_0}\right)N^2 \pm \frac{\omega_p^2}{\omega\Omega} + \frac{\omega_p^2}{\omega^2}\left[K_\perp^2\rho^2\frac{T_\perp}{T_0}J_+\left(\frac{\omega}{|K_\parallel|v_{T_\parallel}}\right) \pm \left(\frac{\Delta T}{T_0}\pm\Omega\right)J_+^{(\alpha)}\left(\frac{\Omega}{|K_\parallel|v_{T_\parallel}}\right)\right]=0 \quad (3.29)$$

Here, we use the following approximate expression for the function /34 $J_+\left(\frac{\omega\mp\Omega}{|K_\parallel|v_{T_\parallel}}\right)$:

$$J\left(\frac{\omega\mp\Omega}{|K_\parallel|v_{T_\parallel}}\right) \approx 1+\frac{K_\parallel^2 v_{T_\parallel}^2}{\Omega^2} \mp J_+^{(\alpha)}\left(\frac{\Omega}{|K_\parallel|v_{T_\parallel}}\right). \quad (3.30)$$

The double sign in Equation (3.29) corresponds to the two roots for the whistlers [see (3.19)]. The solution to Equation (3.29) has the form:

$$\omega = \mp\alpha\Omega + i\gamma, \quad (3.31)$$

where $\alpha = \frac{K^2c^2}{\omega_p^2} \ll 1$, and the instability exponential growth rate γ is:

$$\gamma = \sqrt{\frac{\pi}{2}}\frac{\Omega^2}{|K_\parallel|v_{T_\parallel}}\left[\frac{\Delta T}{T_\parallel} + \alpha\frac{K_\perp^2\rho^2 + \exp\left(-\Omega^2/2K^2v_{T_\parallel}^2\right)}{\alpha K_\perp^2\rho^2 - \exp\left(-\Omega^2/2K^2v_{T_\parallel}^2\right)}\right]\left[\exp\left(-\frac{\Omega^2}{2K^2v_{T_\parallel}^2}\right) - \alpha K_\perp^2\rho^2\right] \quad (3.32)$$

It follows from Equation (3.32) that a small temperature anisotropy leads to unstable oscillations which propagate along the axis in a cone of angle O_0, determined by the relation:

$$\Theta_0^2 = \frac{\omega_p^2}{\Omega^2c^2}\frac{1}{K^2\rho^2}\exp\left(-\frac{T_\perp}{2K^2\rho^2T_\parallel}\right) \ll 1, \quad (3.33)$$

if the condition:

$$\frac{\Delta T}{T_\parallel} > \frac{K^2c^2}{\omega_p^2}. \quad (3.34)$$

is fulfilled.

Since terms proportional to $\frac{\Delta T}{T_\parallel}\frac{\beta_0}{2}\cos^2\Theta$ [see Equation (3.17)] were not considered in the dielectric constant $\varepsilon_{xx}^{(3)}; \varepsilon_{yy}^{(3)}$ in [II], the results obtained above [see Equations (3.18), (3.32)] are more general for the conditions of hydrodynamic and kinetic instabilities.

Turning to the analysis of drift oscillation instabilities ($\omega \sim \omega_{d,M} \ll \omega_0$), we note that in this case the left side of Equation

(3.18) is small for a small degree of anisotropy ($\frac{\Delta T}{T} \ll 1$). Then /35
Equation (3.18) can be reduced to the form:

$$1 - \frac{(\omega - \omega_M)^2 + \omega_M^2}{(\omega - \omega_d)(\omega - \omega_M)} \frac{\Delta T}{T} + \frac{\beta}{2} \frac{(\omega - 2\omega_M)^2}{(\omega - \omega_d)^2} \frac{\Delta T}{T} = 0, \qquad (3.35)$$

where $\omega_d = \frac{k_z v_T^2}{\Omega} \frac{d \ln n}{dx}$, $\omega_M = \frac{k_z v_T^2}{\Omega} \frac{d \ln B}{dx}$. The equilibrium condition $\frac{d}{dx}(nT + B^2/8\pi) = 0$ relates the Larmor frequency (ω_d) and the magnetic drift frequency (ω_M):

$$\frac{\omega_d}{\omega_M} = -\frac{2}{\beta}. \qquad (3.36)$$

If we introduce the variable $x = \frac{\omega - \omega_d}{\omega_d} \frac{T}{\Delta T}$, and consider that Equation (3.35) for $\frac{\Delta T}{T} \ll 1$ can be solved only if $\omega \approx \omega_d$, that is, $\frac{\omega - \omega_d}{\omega_d} \ll 1$, then Equation (3.35) takes the form:

$$x^2 + ax + b = 0 \qquad (3.37)$$

where $a = [(1 + \frac{1}{2}\beta)^2 + \frac{1}{4}\beta^2](1 + \frac{1}{2}\beta)^{-1}$, $b = \frac{1}{2}\beta(1 + \frac{1}{2}\beta)$.

Equation (3.37) has complex roots for $\beta \gtrsim 1$. Thus, drift oscillations ($\omega \sim \omega_d$) are unstable in a plasma with finite pressure ($\beta \gtrsim 1$) for small degrees of anisotropy. The instability exponential growth rate is of the order:

$$\gamma \approx \omega_d \frac{\Delta T}{T}. \qquad (3.38)$$

In a plasma with a sufficiently large pressure ($\beta \gtrsim \frac{T}{\Delta T} \gg 1$), Equation (3.18) reduces to the form:

$$1 - \beta \frac{\Delta T}{T}\left(1 - \frac{1}{2} \operatorname{ctg}^2 \vartheta\right) - \left(\frac{\Delta T}{T}\right)^2 \frac{\beta^3 \omega_d^2}{2(\omega - \omega_d)^2} = 0 \qquad (3.39)$$

in the frequency range $\omega \sim \omega_d$, and has unstable solutions with a frequency $\omega \sim \omega_d$ with a growth rate γ, equal to

$$\gamma \approx \omega_d \frac{\Delta T}{T} \beta \left[\operatorname{ctg}^2 \vartheta - 2\left(1 - \frac{T}{\beta \Delta T}\right)\right]^{-1/2}. \qquad (3.40)$$

It follows from Equation (3.40) that the drift oscillations in an /36
anisotropic plasma with a high pressure ($\beta \gtrsim \frac{T}{\Delta T} \gg 1$) are unstable only

for propagation angles which satisfy the condition:

$$\mathrm{ctg}^2\theta > 2\left(1-\frac{\sim}{\beta_{\alpha T}}\right). \qquad (3.40a)$$

Thus, electron branches of the oscillations can be excited at the first stage of expansion, when the transverse dimension of the cloud is small compared to the ionic Larmor radius. The presence of small pressure (temperature) anisotropies, which are caused by the free cloud expansion along the magnetic field, can lead to a buildup of whistler type oscillations only for a pressure large enough to satisfy the condition $\frac{\Delta T}{T} > 2/\beta$. As the cloud expands, its pressure will fall, and a small degree of anisotropy will not be able to cause a whistler mode instability (if the kinetic instability with an exponentially small increment for $k_\perp \to 0$ is not considered). The electron drift oscillation instability, examined above, is evidently more dangerous at this stage of the cloud evolution.

Drift c) Drift oscillations in an anisotropic plasma magn with magnetized ions ($\beta < 1$)

We now investigate the stability of the drift oscillations in the cloud at the expansion stage when the cloud dimensions become larger than the ionic Larmor radius. In this case, the dielectric constant of both electrons and ions is described by Equations (3.13) to (3.15). We will not examine completely the stability of drift oscillations in a plasma with an anisotropic pressure, but limit ourselves only to explaining the effect of a small pressure anisotropy on the stability of a weakly inhomogeneous plasma with a finite pressure. We consider frequencies and wavelengths contained in the interval: /37

$$k_\parallel v_{T_i} < \omega \sim \omega_d < k_\parallel v_{Te}\ ;\ k_\perp v_{T_i} < \Omega_i .$$

Evaluating terms in the dispersion equation (3.2) shows that purely transverse oscillations occur in the frequency range of interest ($\vec{E} \parallel x,\ k_x = 0$). These oscillations propagate almost perpendicular to the magnetic force lines ($|k_\perp \gg k_\parallel|$), and are described by the equation:

$$1-\frac{\varepsilon_{xx}}{N^2}=1-\beta_{\perp e}\frac{\Delta T_e}{T_{\parallel e}}+\frac{\beta_{\perp i}}{2}\frac{\omega_{di}'}{\omega_{Mi}}C\left(\frac{\omega}{\omega_{Mi}}\right)-$$
$$-i\sqrt{\frac{\pi}{2}}\frac{\omega}{|k_\parallel|v_{T_{\parallel e}}}\beta_{\perp e}\frac{\omega}{|k_\parallel|v_{Te}}\left[\frac{T_{\perp e}}{T_{\parallel e}}-\frac{\omega_{de}}{\omega}-3\frac{\Delta T_e}{T_{\parallel e}}\frac{\omega_{Me}}{\omega}\right]=0; \tag{3.41}$$

where

$$C(a)=\int_0^\infty\frac{x\,dx}{a-x}e^{-x}.$$

These oscillations exist in the plasma only if the degree of anisotropy is not too small:

$$\frac{\Delta T}{T}>\frac{\sqrt{\beta}}{k_\parallel L_\perp, k_\perp L_\perp}. \tag{3.42}$$

Equation (3.41) can be solved analytically only in two frequency regions: 1) $\omega \gg \omega_M$; and 2) $|\omega \ll \omega_M|$. In the first case, the function C(a) is approximately equal to $1/\alpha$, and the solution is easily found to be:

$$\omega\approx-i\sqrt{\frac{\pi}{2}}|k_\parallel|v_{Te_\parallel}\frac{T_{\parallel e}}{T_{\perp e}}\left(1+\beta_{\perp i}-\beta_{\perp e}\frac{\Delta T_{\perp e}}{T_{\parallel e}}\right). \tag{3.43}$$

This solution is valid only for: /38

$$\frac{T_{\parallel e}}{T_{\perp e}}\left|1+\beta_{\perp i}-\beta_{\perp e}\frac{\Delta T_{\perp e}}{T_{\parallel e}}\right|\ll 1, \tag{3.44}$$

The instability occurs for the condition:

$$\frac{\Delta T_{\perp e}}{T_{\parallel e}}>\frac{1+\beta_{\perp i}}{\beta_{\perp e}}. \tag{3.45}$$

Equation (3.44) is satisfied only for $1+\frac{1}{2}\beta_{\perp i}\approx\beta_{\perp e}\frac{\Delta T_e}{T_{\parallel e}}$, that is, it has a resonant character. If it is not satisfied, then the solution to Equation (3.41) will be found in the region $|\omega|\leq\omega_{Mi}$. Assuming for simplicity that $|\omega|\ll\omega_{Mi}$, and using the following approximation for the function C(a) for small $|a|$:

$$C(a)\approx-1+a\{\ln|a|-i[\pi\eta(\mathrm{Re}\,a)\,\mathrm{sgn}\,\mathrm{Im}\,a-\arg a\cdot\mathrm{sgn}\,\mathrm{Re}\,a]\}, \tag{3.46}$$

where:

$$\eta(x)=\begin{cases}1, & x>0\\ 0, & x<0,\end{cases}$$

for the real part of the frequency ω_R leads to the equation:

$$\frac{\omega_R}{\omega_{Mi}}\left(\frac{\omega_R}{\omega_{Mi}} \ln\left|\frac{\omega_R}{\omega_{Mi}}\right| - \frac{1}{2}\beta_{\perp i}\right) = \beta_{\perp e}\frac{\Delta T}{T_{\parallel e}}. \qquad (3.47)$$

If we limit ourselves to the case of small anisotropies and take $\frac{T_{\perp e}}{T_{\perp i}}\frac{\Delta T_e}{T_{\parallel e}} \ll 1$, and if we are not interested in large pressures ($\beta_{\perp e} \lesssim 1$), then Equation (3.47) has a solution in this frequency range $\omega \ll \omega_{Mi}$, which is approximately written in the form:

$$\omega_R = \omega_{Mi}\, a_0 \equiv -2\omega_{Mi}\frac{T_{\perp e}}{T_{\perp i}}\frac{\Delta T_e}{T_{\parallel e}} \qquad (3.48)$$

Evaluation of the anti-Hermitian part of (3.41) yields the following expression for the instability increment:

$$\gamma \simeq \pi\left[2\frac{\omega_{\perp e}\,\omega_{Mi}}{|k_\parallel| v_{Te}} - \eta(-\Delta T_e)\, a_0^2\,|\omega_{Mi}|\left(a_0 + \frac{2}{\beta_{\perp i}}\right)\right] \qquad (3.49)$$

Equation (3.49) shows that unstable oscillations with a frequency less than the magnetic drift frequency can exist in an anisotropic weakly inhomogeneous plasma if the temperature anisotropy and the oscillation wavelength are related by the equation: /39

$$1 > \frac{\Delta T_e}{T_{\parallel e}} > -\frac{k_\perp \rho_e}{k_\parallel L}. \qquad (3.50)$$

4. Effect of the External Plasma (Medium) on the Cloud Stability ($\beta \lesssim 1$)

The previous section examined the stability of the cloud at the stage of its motion for $\beta \lesssim 1$ when the presence of the surrounding magnetospheric plasma was neglected. Here, the effect of the external plasma will be considered on the anisotropic instability of the cloud. For convenience, the investigation will be done in a system which is at rest with the investigated cloud element. In this system, external plasma particles fly into the cloud with a velocity $v_0 \equiv u$ along the field, and with a velocity v^0 perpendicular to the field (see section 1). We consider the case when the average particle velocity is along the field, that is $\vec{u} = u\vec{e}_z$. This is valid, for example, at the stage of cloud motion when the transverse cloud dimensions become larger than the ionic Larmor radius v/Ω_i for the rarefied magnetospheric plasma, which flies into the cloud ($\bar{\beta} \ll 1$; parameters with a line above them refer to the external plasma). Here, the drift in the crossed geomagnetic and electric fields of the polarized cloud will set the external plasma particles in motion with the same direction and velocity as those of the cloud as a whole (the cloud injection velocity $\vec{v}_0$). Thus, the average transverse motion of the magnetospheric plasma can be neglected relative to that of the cloud. The presence of a relative average velocity between the plasmas of the magnetosphere and the cloud can be related only with the expansion of the cloud along the magnetic field force lines. Here we note that the case of cloud injection along the geometric field is of definite interest. /40

For cloud parameters of interest, the magnetospheric plasma can be considered cold ($\bar{T} \ll T$) and rarefied ($\bar{n} \ll n$) in relation to the cloud plasma. We consider the stability of oscillations in the plasma for which electrons can be considered to be magnetized $\Omega \gg \omega,\ \omega',\ \kappa v_T$, ($\omega' = \omega - \kappa_\parallel u$), and take $\omega \gg \Omega_i,\ \kappa v_{Ti}$, and $\omega' \gg \bar{\Omega}_i,\ \kappa \bar{v}_T$ for the ions. In the frequency region $\omega \ll \kappa_\parallel v_{T\parallel}$, $\omega' \gg \kappa_\parallel \bar{v}_T$, both a small degree of plasma anisotropy ($\Delta = \frac{\Delta T}{T_\perp} \lesssim 1$) and the plasma current velocity $\frac{u}{v_T} \lesssim 1$ have a small effect on the plasma stability, and the exponential growth rate can be rather large. In this region, the contribution of the ions* to the dielectric constant tensor can be neglected. The components of this tensor can be written in the form (in a coordinate system where $\kappa_\perp = \kappa_x$):

---

*This is correct, if we do not consider the case of wave propagation too close to the direction $\theta = \pi/2$ (which is related, of course, to the whistlers examined in section 3).

$$
\begin{aligned}
&\varepsilon_{xx} = -\frac{\omega_p^2}{\Omega^2}\frac{k_\parallel^2 v_{T_\perp}^2}{\omega^2}\left(\Delta - \frac{\bar{n}}{n}\frac{\omega'^2}{k_\parallel^2 v_{T_\parallel}^2}\right)\\
&\varepsilon_{yy} = -\frac{\omega_p^2}{\Omega^2}\frac{k_\parallel^2 v_{T_\parallel}^2}{\omega^2}\left[\Delta\left(1 - 2\frac{T_\perp}{T_\parallel}\mathrm{tg}^2\theta\right) + 2\frac{T_\perp^2}{T_\parallel^2}\mathrm{tg}^2\theta J_+ - \frac{\bar{n}}{n}\frac{\omega'^2}{k_\parallel^2 v_{T_\parallel}^2}\right]\\
&\varepsilon_{zz} = -\frac{\omega_p^2}{\Omega^2}\frac{k_\parallel^2 v_{T_\perp}^2}{\omega^2}\left[\Delta\,\mathrm{tg}^2\theta - \frac{\Omega^2\omega^2}{k_\parallel^4 v_{T_\parallel}^4}\left(1 - J_+ - \frac{\bar{n}}{n}\frac{k_\parallel^2 v_{T_\perp}^2}{\omega'^2}\right) - \frac{\bar{n}}{n}\frac{u^2}{v_{T_\perp}^2}\mathrm{tg}^2\theta\right]\\
&\varepsilon_{xy} = -\varepsilon_{yx} = i\frac{\omega_p^2}{\omega\Omega}\left(1 + \frac{\bar{n}}{n}\frac{\omega'}{\omega}\right)\\
&\varepsilon_{xz} = \frac{\omega_p^2}{\Omega^2}\frac{k_\parallel^2 v_{T_\parallel}^2}{\omega^2}\,\mathrm{tg}\,\theta\left(\Delta - \frac{\bar{n}}{n}\frac{u}{v_{T_\parallel}}\frac{\omega'}{k_\parallel v_{T_\parallel}}\right)\\
&\varepsilon_{yz} = -\varepsilon_{zy} = i\frac{\omega_p^2}{\omega\Omega}\mathrm{tg}\,\theta\left[\frac{T_\perp}{T_\parallel}(1 - J_+) - \frac{\bar{n}}{n}\frac{k_\parallel u}{\omega}\right]
\end{aligned}
\qquad (4.1)
$$

Substituting the tensor ε_{ij} from Equation (4.1) and performing /41
certain transformations reduce the dispersion equation (3.2) to the form (if small terms $\sim J_+\left(\frac{\omega}{|k_\parallel| v_{T_\parallel}}\right)$ are neglected):

$$
\begin{aligned}
&\left(\Delta + \frac{2}{\beta_\parallel} - \frac{\bar{n}}{n}\frac{u^2}{v_{T_\parallel}^2}\right)\left[\Delta + \frac{2}{\beta_\parallel} - \frac{\bar{n}}{n}\frac{u^2}{v_{T_\parallel}^2} + 2\,\mathrm{tg}^2\theta\left(\frac{1}{\beta_\parallel} - \Delta\frac{\Delta+2}{2}\right)\right] -\\
&- \frac{\bar{n}}{n}\frac{k_\parallel^2 v_{T_\parallel}^2}{\omega'^2}\left(\Delta + \frac{2}{\beta_\parallel}\right)\left\{\Delta + \frac{2}{\beta_\parallel} - \frac{\bar{n}}{n}\frac{u^2}{v_{T_\parallel}^2} + 2\,\mathrm{tg}^2\theta\left[\frac{1}{\beta_\parallel} - \Delta(\Delta+1)\right]\right\} -\\
&- \frac{\omega^2\Omega^2}{k_\parallel^4 v_{T_\parallel}^4}\left(1 - \frac{\bar{n}}{n}\frac{k_\parallel^2 v_{T_\parallel}^2}{\omega'^2}\right)\left(1 - \frac{\bar{n}}{n}\frac{k_\parallel u}{\omega}\right)^2 = 0
\end{aligned}
\qquad (4.2)
$$

In the case $\bar{n} \to 0$, Equation 4.2 transformed into the spectral Equation (3.19). This is a fourth-order equation; in general, it is difficult to solve it. First of all, we note that we are only analyzing the stability of plasmas which have a small degree of anisotropy ($\Delta \ll 1$) and directional velocity $\left(1 \gg \bar{\beta}_u = \frac{8\pi\bar{n}mu^2}{B^2} = \frac{\bar{n}}{n}\frac{u^2}{v_T^2}\beta\right)$

For long-wavelength oscillations $k\rho_e \ll \frac{u}{v_T}\beta$, unstable solutions to /42
Equation (4.2) lie in the frequency region $\omega \ll k_\parallel u$, and have the form:

$$
\omega = \frac{\bar{n}}{n}k_\parallel u \pm i\Omega\frac{kk_\parallel c^2}{\omega_p^2}\left(1 + \frac{\Delta}{2}\beta_\parallel - \frac{1}{2}\beta_u\right)\sqrt{Q\sin^2\theta - 1}, \qquad (4.3)
$$

where

$$
Q = \frac{\Delta^2\beta_\parallel\frac{nu^2 - 2\bar{n}v_T^2}{nu^2 - \bar{n}v_T^2} + 3\Delta\beta_\parallel - \beta_u}{2 + \Delta\beta_\parallel - \beta_u},
$$

and

$$|n u^2 - \bar{n} v_{T_\parallel}^2| \gg \frac{\bar{n}^2 u^2 \beta_\parallel}{n|2 + \Delta\beta_\parallel|}$$

The instability condition is

$$Q \sin^2\theta > 1 . \tag{4.4}$$

In the case of a large velocity "u", when $\tilde{\beta}_u \gtrsim 1$ (that is, $u \gg v_T$), the condition for oscillation buildup (4.4) is governed by the inequality

$$\tilde{\beta}_{min} < \tilde{\beta}_u < \tilde{\beta}_{max} , \tag{4.5}$$

where

$$\tilde{\beta}_{\substack{min \\ max}} = \substack{min \\ max} \left\{ 2 + \Delta\beta_\parallel ; \frac{2 + \Delta\beta_\parallel - \Delta(\Delta+3)\beta_\parallel \sin^2\theta}{\cos^2\theta} \right\} .$$

In particular, the instability occurs even if the temperature anisotropy is neglected, that is, for $(1+\Delta)\Delta\beta_\parallel \ll 1$. Here the condition for a purely current instability is the inequality:

$$2 < \tilde{\beta}_u < \frac{2}{\cos^2\theta} . \tag{4.6}$$

If the velocity of directional pressure is small ($\tilde{\beta}_u \ll 1$), then the instability condition [Equation (4.4)] has the form: /43

$$\Delta^2 p \sin^2\theta + \Delta(3\sin^2\theta - 1) - \frac{2}{\beta_\parallel} > 0 , \tag{4.7}$$

where

$$p = \frac{1 - 2\frac{\bar{n}}{n}\frac{v_{T_\parallel}^2}{u^2}}{1 - \frac{\bar{n}}{n}\frac{v_{T_\parallel}^2}{u^2}} .$$

The Inequality (4.7) can be fulfilled only in the presence of a temperature anisotropy in the cloud ($\Delta \neq 0$).

Condition (4.7) is fulfilled for (we are mainly interested in the case $\Delta > 0$):

$$\begin{aligned} &\Delta > \Delta_+ , \quad (p > 0) \\ &\Delta_+ < \Delta < \Delta_- , \quad (p < 0) ; \end{aligned} \tag{4.8}$$

where:

$$\Delta_{\pm} = \frac{1-3\sin^2\vartheta \pm d^{1/2}}{2p\sin^2\vartheta}, \quad d = (1-3\sin^2\vartheta)^2 + \frac{8p}{\beta_\parallel}\sin^2\vartheta$$

The instability occurs only for d > 0, that is, for:

$$p > p^* = -\frac{\beta_\parallel}{8}\frac{(1-3\sin^2\vartheta)}{\sin^2\vartheta} \qquad (4.9)$$

For $\tilde{n} \to 0\ (\frac{\tilde{n}}{n}\frac{v_{T_n}^2}{U^2} \ll 1)$, when $p \to 1$, Equation (4.8) transforms into the instability conditions given by Equation (3.21), above, for the case $\sin^2\vartheta = 1,\ \Delta > 0$. We investigate how the change in the parameter $\nu = \frac{\tilde{n}}{n}\frac{v_{T_n}^2}{U^2}$ affects the plasma stability. The parameter ν should increase monotonically as the cloud diffuses, because in all probability, the cloud density will decrease faster than its temperature. As ν increases from zero to $\nu = \nu^* = \frac{1-p^*}{2-p^*} < 1$, the parameter p decreases from 1 to p*. Here the minimum value of the degree of anisotropy (Δ_+) increases correspondingly from a value $\Delta_1 = \Delta_+(p=1)$ to $\Delta^* = \Delta_+(p=p^*)$, above which the instability condition (Δ_+) is satisfied (see Figure 2). Furthermore, instability occurs in the interval $\nu^* < \nu < 1$, where $p < p^*$. For ν > 1, the quantity Δ_+ sharply falls and then monotonically rises to the value $\Delta_2 = \Delta_+(p=2)$, with further growth of ν. The value of Δ_+ at p = 2 is much smaller than the value at p = 1. We note that the value of p is bounded from above (ν should not be close to 1) by the observability condition $(\omega \ll \kappa_\parallel u)$. Thus, for $\frac{\tilde{n}}{n} > \frac{U^2}{v_{T_n}^2}\ (\nu > 1)$, the observed anisotropic instability can develop for a smaller degree of temperature anisotropy in the cloud than for the case $\frac{\tilde{n}}{n} < \frac{U^2}{v_{T_n}^2}\ (\nu < 1)$. It follows from Equation (4.3) that the corresponding exponential growth rate for $\frac{\tilde{n}}{n} > \frac{U^2}{v_{T_n}^2}$ is larger than in the opposite case (remaining parameters being the same). The unstable perturbations are distributed in a cone with angles $\vartheta_{\kappa p} < |\vartheta| < \pi - \vartheta_{\kappa p}$, where $\vartheta_{\kappa p} = \arcsin\frac{1}{\sqrt{\Theta}}$. As ϑ increases in the interval $\vartheta_{\kappa p} < \vartheta < \frac{\pi}{2}$, the growth rate initially increases from zero to a maximum value $\gamma(\vartheta_m)$ for $\vartheta = \vartheta_m = \arcsin\sqrt{\frac{\Theta+1}{2\Theta}}$, and then falls. It follows from /44

Equation (4.3) that the exponential rate for perturbation growth in the system of cloud + plasma is of the same order as the increment for the buildup of the whistler mode for the cloud:

$$\gamma \sim \frac{k^2 v_T^2}{\beta |\Omega|}. \tag{4.10}$$

The maximum value of the growth rate is determined by the observability condition $\gamma \ll k_\parallel v_T, k_\parallel u \ll \Omega$, that is, it can attain values of 0.01 to 0.1 megahertz (for $u \sim 0.1 v_T$). /45

Unstable perturbations with shorter wavelengths ($k\rho_e \gg \frac{u}{v_T}\beta$) have a phase velocity which is larger than the current velocity ($\frac{\omega}{k_\parallel} \gg u$). Because of the observability condition $k\rho_e \ll 1$, $\beta \lesssim 1$, the directional velocity of the current u should be small compared to the thermal velocity of the electrons v_T. Here the parameter $\bar{\beta}_n$ is small, and the temperature anisotropy ($\Delta \neq 0$) of the cloud plasma significantly affects the instability.

If the perturbations have a sufficiently small wavelength ($k\rho_e \gg \frac{u}{v_T}\beta$), when the frequency spectrum lies only in the region $\omega \gg k_\parallel u$, then Equation (4.2) becomes biquadratic, and has the solutions:

$$\omega^2 = k_\parallel^2 v_{T_\parallel}^2 \frac{k_\parallel^2 v_{T_\parallel}^2}{2\Omega^2} (\mu + a_1)\left(1 \pm \sqrt{1 - 4\frac{\mu a_2}{(\mu + a_1)^2}}\right), \tag{4.11}$$

where

$$\mu = \frac{\bar{n}}{n} \frac{\Omega^2}{k_\parallel^4 v_{T_\parallel}^2}, \quad a_1 = a_2 + \Delta^2 tg^2 \vartheta \left(\Delta + \frac{2}{\beta_\parallel}\right),$$
$$a_2 = \left(\Delta + \frac{2}{\beta_\parallel}\right)\left\{\Delta + \frac{2}{\beta_\parallel} + 2 tg^2 \vartheta \left[\frac{1}{\beta_\parallel} - \Delta(\Delta + 1)\right]\right\}.$$

From Equation (4.11), it can be seen that, along with the whistler-mode distortions (3.19), the presence of the external rarefied (magnetospheric) plasma causes a new spectrum of oscillations which are absent for $\bar{n} = 0$. The oscillation spectrum (4.11) is stable only if the conditions

$$
\left.\begin{array}{l} a_2 > 0 \\ a_1 + \mu > 0 . \end{array}\right| \tag{4.12}
$$

are satisfied simultaneously. Thus, the development of perturbation instabilities requires that at least the condition $a_2 < 0$ be satisfied, or else $\Delta > \Delta_2$, which is weaker than the instability condition (3.20) $a_1 < 0$ (or $\Delta > \Delta_1 > 0$), for the whistler mode (3.19), but stronger than the condition $\Delta > \Delta_+$ for perturbation instabilities with $\omega \ll k_\parallel u$ for $\frac{\bar{n}}{n} > \frac{u^2}{v_{T\parallel}^2}$. For $\kappa\rho_e \leq \beta\left(\frac{\bar{n}}{n}\right)^{\frac{1}{2}}$, the oscillation spectrum (4.11) is unstable for $\Delta > \Delta_2$ and has a growth rate on the order of

$$
\gamma \sim \frac{|k_\parallel| v_{T\parallel}\left(\frac{\bar{n}}{n}\right)^{\frac{1}{2}}}{1 + \frac{\beta}{\kappa\rho_e}\left(\frac{\bar{n}}{n}\right)^{\frac{1}{2}}} . \tag{4.13}
$$

If $\kappa\rho_e \gg \beta\left(\frac{\bar{n}}{n}\right)^{\frac{1}{2}}$, then the growth rate can be significantly larger than Equation (4.13). In this case, only the whistler-mode occurs from the spectral equation (4.11). This mode is unstable (3.19) for $\Delta > \Delta_1$ and has a growth rate given by Equation (4.10). A new mode also occurs, which is unstable for $\Delta_2 < \Delta < \Delta$, and has the growth rate $\gamma \sim |k_\parallel| v_{T\parallel}\left(\frac{\bar{n}}{n}\right)^{\frac{1}{2}}$. This last oscillation branch exists in the region $\omega \gg k_\parallel u$ for $\nu = \frac{\bar{n}}{n}\frac{v_{T\parallel}^2}{u^2} \gg 1$, and transforms into the region $\omega \sim k_\parallel u$ for $\nu \ll 1$.

Finally, we consider other possible solutions to Equation (4.2). in the region $\omega \sim k_\parallel u$. The wavelength is of the order $\kappa\rho_e \sim \beta\frac{u}{v_T}$ for these unstable oscillations. Here, if $\nu \gg 1$, then the oscillation frequency is equal to:

$$
\omega = \pm k_\parallel v_{T\parallel} \frac{k_\parallel v_{T\parallel}}{\Omega} a_2^{1/2} \tag{4.14}
$$

The instability growth rate (for $\Delta > \Delta_2$, when $a_2 < 0$) is of the same order as the rate for the whistler mode (3.19). If $\nu \ll 1$, then the oscillation spectrum reduces to a pure whistler mode, an unstable mode $\Delta > \Delta_1$, and a new mode:

$$
\omega = k_\parallel u\left(1 \pm \sqrt{\nu\frac{\mu + a_1\nu}{\mu - a_1\nu}}\right), \tag{4.15}
$$

which is unstable for the condition /47

$$a_2 \nu < \mu < a_1 \nu \equiv a_2 \nu + \nu' \Delta^2 \left(\Delta + \frac{2}{\beta_n}\right) tg^2 \vartheta. \qquad (4.16)$$

It follows from this that the frequency instability (4.15) ($\kappa_\parallel u \approx \omega$) is also possible for small degrees of cloud temperature anisotropy (Δ). The exponential growth rate is of the order

$$\gamma \sim |\kappa_\parallel| v_{T_0} \left(\frac{\bar{n}}{n}\right)^{1/2} \sim |\Omega \cos \vartheta| \frac{u}{v_T} \beta \left(\frac{\bar{n}}{n}\right)^{1/2}. \qquad (4.17)$$

Thus, in the frequency region $\omega \gg \kappa_\parallel u$, the maximum rate is actually of the same order as in the region $\omega \ll \kappa_\parallel u$, that is, of the same order as the rate for the buildup of the whistler mode.

Conclusions

Analysis of the stability of an anisotropic inhomogeneous cloud /48
plasma, which propagates in the magnetosphere along the magnetic field force lines, produced the following results. If the degree of cloud temperature anisotropy, which results from its expansion, is small ($\Delta = \frac{T_\perp - T_\parallel}{T} \leq 1$), then the hydrodynamic whistler instability will be substantial only for the initial stage of diffusional expansion. Here, $\beta \gg 1$ and the condition $\Delta > \frac{2}{\beta}$ is fulfilled for the hydromagnetic whistler instability. The characteristic rise time for these oscillations is approximately 10^{-5} seconds. As the cloud expands further, the kinetic whistler instability ($\kappa_\perp = 0$) becomes more dangerous. This instability has an exponential growth rate* $\gamma \sim 10^3$ sec^{-1}, for $\Delta \sim 1$ and $\kappa^2 \rho_e^2 \sim 0.1$.

Considering the cloud plasma inhomogeniety in the presence of a temperature anisotropy leads to the discovery of new branches of unstable drift oscillations. At the initial stage of diffusional

*A rather complete analysis of stability of whistlers, which propagate along the magnetic field ($\kappa_\perp = 0$), is given in [12], for example.

expansion, when ($\rho_i > L$), an instability occurs with an exponential growth rate on the order of frequency for drift electron oscillations:

$$\omega \sim \gamma \sim k v_T \frac{\rho_a}{L} > \frac{\rho_e v_{Te}}{L^2} \sim 10^4 \sec^{-1},$$

i.e., this rate exceeds the rate for the instability for kinetic whistler oscillations. Moreover, as opposed to the whistlers, which propagate almost entirely along the magnetic field force lines ($k_L = 0$), these perturbations do not have a rigorously separate direction, that is, they can be recorded in a direction perpendicular to the magnetic field. Further expansion of the cloud leads to unstable drift oscillations with a frequency, /49

$$\omega_* \sim \gamma \sim \frac{\rho_i v_{Ti}}{L^2} \sim 10 \sec^{-1},$$

when the ions also become magnetized ($\rho_i < L$, $L \gtrsim 10^2 м$).

The following statements can be made about the development of anisotropic current instabilities, which were examined in this paper. First we note that perturbations with large phase velocities $\frac{\omega}{k_\parallel} \gg u$ can exist when the spreading velocity of the cloud along the force lines is small compared to the thermal velocity of the cloud electrons (v_{Te}), because of the observability conditions $k v_{Ti} \ll \omega \ll k_\parallel v_{Te}$, $\omega \gg k_\parallel \tilde{v}_{Te}$. Here, the ratios $\frac{u}{v_{Ti}}$ and $\frac{u}{\tilde{v}_{Te}}$ are arbitrary. Perturbations with smaller phase velocities $\frac{\omega}{k_\parallel} \lesssim u$ can grow in the plasma only for a sufficiently large flow velocity $u \gg v_{Ti} + \tilde{v}_{Te}$. Thus, if conditions $u \ll v_{Te}$, $\Delta \sim 1$ are satisfied, then (for $1 > \Delta$) short wave-length perturbations for which $(k\rho_e)^2 \gg \frac{\tilde{n}}{n} + \frac{u^2}{v_{Te}^2}$ will grow at the initial stage of diffusional cloud expansion ($\beta \sim 1$, $\frac{\tilde{n}}{n} \ll 1$) with a maximum exponential rate [on the order of the whistler mode rate from Equation (4.10), but with $\omega \gg k_\parallel u$]:

$$\gamma_m \sim |\Omega_e|(\kappa\rho_e)^2$$

The rise time of this instability can attain values of up to 10 times the cyclotron period, that is, up to 10^{-5} sec^{-1} (with a consideration of the conditions $\omega \ll \Omega_e$ and $\kappa\rho_e \ll 1$). In the case of $\Delta < \Delta_1$, the growth rate is much less. At the subsequent stage of cloud expansion, when β becomes small (as previously, $\bar{n}/n \ll 1$), /50 such that $\Delta \ll \beta^{-1}$ (or if at the stage $\beta \sim 1$ the cloud temperature anisotropy is small), the instability has a resonant character with respect to the wavelength

$$(\kappa\rho_e)^2 \approx (\beta \frac{u}{v_{Te}})^2 \gg \beta^2 \frac{\bar{n}}{n}$$

and has an exponential growth rate $\gamma \sim |\Omega|\beta \frac{u}{v_{Te}} (\frac{\bar{n}}{n})^{\frac{1}{2}} \ll \gamma_m$. [For $\omega \approx \kappa_\parallel u$, see Equation (4.17).] Here, the condition $u^2 \gg v_{Ti}^2 + \bar{v}_{Te}^2 + v_{Te}^2 \frac{\bar{n}}{n}$ must be fulfilled. This instability decreases as the cloud continues to expand, when its density falls to a value of $n \lesssim \bar{n} \frac{v_{Te}^2}{u^2}$. Thence it follows that if the flow velocity of the cloud along the magnetic field is small $u \lesssim v_{Ti}$ (but $u \gg \bar{v}_{Te}$), then instability is possible for a small degree of cloud anisotropy ($\Delta \lesssim 1$) only in the initial diffusional expansion stage ($\beta \sim 1$). For all the parameters chosen for evaluation in this article, the maximum cloud expansion velocity along the geomagnetic field only slightly exceeds the ionic thermal velocity in the case of a hydrogen plasma, but is much larger than the ionic thermal velocity for a barium plasma. Thus, if the injected plasma cloud does not have a significant initial temperature anisotropy, and if the ions are not able to cool significantly during the initial cloud expansion stage ($\beta \gtrsim 1$), then a hydrogen plasma could be unstable relative to the hydrodynamic current instability only during the initial diffusional expansion stage ($\beta \sim 1$), while a barium plasma can remain unstable for further cloud fusion, as long as the relationship $n > \bar{n} \frac{v_{Te}}{u^2}$ is satisfied.

The above linear stability analysis of oscillations of an anisotropic inhomogeneous plasma, which flows along the magnetic field force lines in the magnetosphere, does not answer the question of the physical consequences of one instability or another. These /51 answers require a study of the nonlinear stages of instability development. However, the following statements can be made relative to the drift instabilities of an inhomogeneous plasma which has a temperature anisotropy. As in the case of a convection current instability, which is also related to the inhomogeneity and particle velocity distribution anisotropy, these instabilities should lead to additional convection, that is, to equalization of the kinetic and magnetic pressure gradients. In the final analysis, this can lead to a faster diffusion expansion of the cloud. As the instabilities develop, which are related to the temperature anisotropy, there is a tendency to weaken the temperature anisotropy, although continued longitudinal expansion of the cloud will somewhat maintain the cause of the instability.

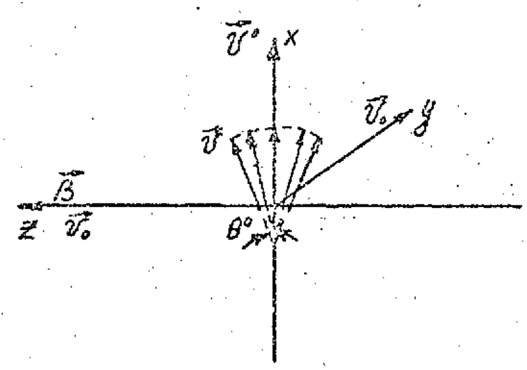

Figure 1. Injection of a plasma cloud at the initial moment of time

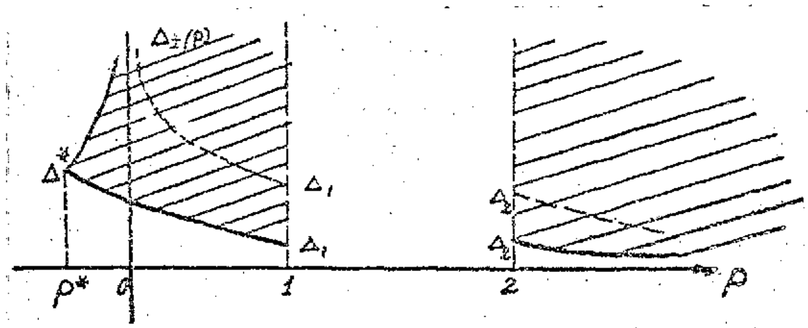

Figure 2. Boundaries of the instability region (shaded). The broken curve corresponds to the condition $|\sin\vartheta| < \frac{1}{\sqrt{3}}$. The solid curve corresponds to $|\sin\vartheta| > \frac{1}{\sqrt{3}}$.

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